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A Class of the Hamming Weight Hierarchy of Linear Codes with Dimension 5

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Abstract: The weight hierarchy of a $[n,k;q]$ linear code C over F_q is the sequence $(d_1,\cdots,d_r,\cdots,d_k)$, where d_r is the smallest support weight of an r-dimensional subcode of C. In this paper, by using the finite projective geometry method, we research a class of weight hierarchy of linear codes with dimension 5. We first find some new pre-conditions of this class. Then we divide its weight hierarchies into six subclasses, and research one subclass to determine nearly all the weight hierarchies of this subclass of weight hierarchies of linear codes with dimension 5.

Key words: generalized Hamming weight; weight hierarchy; linear code; difference sequence; finite projective geometry

1 Introduction

Information theory is the theory of communication systems, and coding theory is an important part of information theory. Hamming weight was proposed by Hamming[1]. In 1991, Wei[2] proposed the generalized Hamming weight which described the cryptography features of linear codes.

For a code D, the support set[2] of D is expressed as $\chi(D)$, that is

$$\chi(D)=\bigcup_{c\in D}\{i|c_i\neq 0\,,c=(c_1,c_2,\cdots,c_n)\}.$$

The size of the support set is denoted by $\omega_s(D)=|\chi(D)|$.

For a $[n,k;q]$ code C and $1\leqslant r\leqslant k$, the r-th generalized Hamming weight (or the minimum support weight) is defined by

$$d_r(C)=\min\{\omega_s(D)|D \text{ is a } [n,r;q] \text{ subcode of } C\}$$

and in particular, $d_1(C)$ is the minimum distance of C. The sequence (d_1,d_2,d_3,d_4,d_5) is the generalized Hamming weight or weight hierarchy for short.

Another form of weight hierarchy is called length/dimension outline by Forney[3]. The weight hierarchy has important applications in the trellis complexity analysis of linear codes[3-7], in decoding analysis[8-10], and in error detection analysis[11]. In Ref. [12], the applications of weight hierarchy are discussed in detail. Since Wei[2] proposed the concept of weight hierarchy, this concept has played an increasingly important role in coding theory. In Refs. [13-16], the upper and lower bounds were researched. In Refs. [17, 18], weight hierarchy was expanded to linear constant weight codes. We also refer Refs. [19, 20] in this paper.

1.1 Related work

In Ref. [21], "determining all possible weight hierarchies of general linear codes" was proposed, and it remains a significant theoretical problem in coding theory. In 1996, Chen and Kl∅ve proposed the finite projective geometry method, and this method was first effectively used to determine the weight hierarchies of q-ary linear codes with dimension 4[22]. For years, research on weight hierarchies of linear codes has made significant achievements[23,24]. In Ref. [6], q-

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ary linear codes and weight hierarchies of dimension 4 were divided into nine types, and some of these types were researched by using the finite projective geometry method[23]. In Ref. [25], q-ary linear codes and weight hierarchies with dimension 5 were divided into six types, and type II-1 was researched. In Ref. [26], type II-2 was determined. In Ref. [27], almost all weight hierarchies of type V-1 were determined. And in Ref. [28], almost all weight hierarchies of type V-2 were determined. The remaining three types of weight hierarchies of q-ary linear codes with dimension 5 are as yet undetermined, because either the upper or lower bound is greater than one, making them difficult to research.

1.2 Main contribution

Of the identified six types[25] of weight hierarchies of q-ary linear codes with dimension 5, in this study we research type VI. A new difficulty in researching type VI is that there is more than one lower bound. Therefore type VI must be divided into smaller subclasses. By using the finite projective geometry method, we identify a number of new necessary conditions. By these conditions, the weight hierarchies of type VI can be divided into six small subclasses. By using the finite projective geometry method and the subspace set method proposed by Ref. [29], we determine almost all possible weight hierarchies of type VI-1 of q-ary linear codes with dimension 5.

1.3 Article structure

In this paper, we investigate the type VI weight hierarchies proposed in Ref. [25]. In Section 2, we introduce some background knowledge of the weight hierarchy of linear codes. Theorem 1 is the necessary condition of type VI, as proposed in Ref. [25]. In order to gain almost all the weight hierarchies, we must prove that the necessary condition is almost sufficient. In Sections 3 and 4 we provide our main results. In Section 3, we describe some new necessary conditions of type VI, and using these we divide type VI into six classes (see Theorem 2). In Section 4, we research the first subclass of type VI (VI-1). Lemmas 1-5 are constructions and proofs under different conditions. Using Lemmas 1-5, we get almost all the weight hierarchies of type VI-1. Finally with Theorem 3, we prove that the necessary condition of type VI-1 is almost sufficient. Thus we gain almost all the possible weight hierarchies of type VI-1. In Section 5, we summarize our main results.

2 Preparation

Unless otherwise stated, C will be a $[n, 5; q]$ linear code; that is, a linear code with length n and dimension 5 over F_q. The difference sequence $(d_1, d_2, d_3, d_4, d_5)$ is the weight hierarchy of C.

Code C corresponds to the sequence $(d_1, d_2, d_3, d_4, d_5)$. All linear codes C confirm a sequence set. Our goal is to determine almost all sequences in this sequence set.

We note that if we add a zero-position to $[n, 5; q]$ code C, we get a $[n+1, 5; q]$ code $C' = \{(c|0)|c \in C\}$. The codes C and C' have the same weight hierarchy. Therefore, we can restrict ourselves to codes without a zero-position; that is, we will assume that $n = d_5$.

The Difference Sequence (DS) $(i_0, i_1, i_2, i_3, i_4)$ of a $[d_5, 5; q]$ code C is defined by

$i_0 = d_5 - d_4, i_1 = d_4 - d_3, i_2 = d_3 - d_2,$
$i_3 = d_2 - d_1, i_4 = d_1.$

Obviously DS can be obtained by the weight hierarchy, and vice versa. Therefore, "determining the weight hierarchy" is equivalent to "determining the difference sequence". It was shown in Ref. [2] that $i_r \geqslant 1$ for all r.

Let $\boldsymbol{G}$ be a generator matrix for C. For $\forall \boldsymbol{x} \in F_{q^5}$, $m(\boldsymbol{x})$, the value of $\boldsymbol{x}$ will denote the number of occurrences of $\boldsymbol{x}$ as a column in $\boldsymbol{G}$. If $\boldsymbol{y}$ is a column in $\boldsymbol{G}$, and $\boldsymbol{x} = \alpha \boldsymbol{y}$ for some nonzero $\forall \boldsymbol{x} \in F_{q^5}$, we may replace $\boldsymbol{y}$ with $\boldsymbol{x}$ without changing the support weight of any subcode. Therefore, we may view the vectors as points in the projective space $V_4 = PG(4, q)$. A value assignment is the function

$$m : V_4 \to N, N = \{0, 1, 2, \cdots\}.$$

For $\forall p \in V_4$, we call $m(p)$ the value of p (or weight). For $\forall S \in V_4$, we define the value of a subset S as $m(S) = \sum_{p \in S} m(p)$.

In Ref. [5], it was proposed that the existence of a code with the weight hierarchy $(d_1, d_2, d_3, d_4, d_5)$ is equivalent to the existence of a value assignment m such that

$$\max\{m(U_r)|U_r \text{ is a subspace with dimension } r \text{ in } V_4\} = \sum_{j=0}^{r} i_j, 0 \leqslant r \leqslant 4 \tag{1}$$

Let p^*, l^*, P^*, and V^* be the heaviest point, the heaviest line, the heaviest plane, and the heaviest body satisfying Formula (1), respectively when $r = 0, 1, 2, 3$. The main reason for using the finite projective geometry method to determine almost all the weight hierarchies of linear codes is that it yields the tightest and best necessary conditions. We can then construct the function m, and even as far as possible satisfy formal Formula (1) according to almost all i_j, thus satisfying this necessary condition.

Definition 1 Let $N(i)$ be the number of difference sequences of some type with $i_0 \leqslant i$, and let $M(i)$ be the number of sequences satisfying the necessary condition of the same type with $i_0 \leqslant i$. If $\lim\limits_{i\to+\infty}\dfrac{N(i)}{M(i)} = 1$, we call the necessary condition almost sufficient.

As stated above, the division of the necessary conditions of the difference sequences of q-ary linear codes with dimension 5 into six classes by Ref. [25] was equivalent to classifying the weight hierarchies into six classes. This paper investigates type VI.

Theorem 1 For q-ary linear codes with dimension 5, the necessary conditions[25] for almost all sequences $(i_0, i_1, i_2, i_3, i_4)$ to be class VI difference sequences are

(1) $i_1 \leqslant qi_0$;

(2) $qi_1 < i_2 \leqslant \dfrac{q^2}{q+1}(i_0 + i_1)$;

(3) $qi_2 < i_3 \leqslant \dfrac{q^3}{q^2+q+1}(i_0 + i_1 + i_2)$;

(4) $\max\{i_0, i_1\} \leqslant i_4 \leqslant (q^3 + q^2 + q)i_1 - i_2 - i_3$.

3 New Necessary Conditions and Classification of Type VI

In order to prove Theorem 1, we must find the sufficient conditions of type VI, and these sufficient conditions are very close to being necessary conditions. We next infer some key necessary conditions.

If $p^* \in P^*$, we can get

$$m(P^*) = i_0 + i_1 + i_2 = i_0 + \sum_{p^*\in l\subset P^*}(m(l) - i_0) \leqslant i_0 + (q+1)i_1.$$

So $i_2 \leqslant qi_1$, which is contradictory to $i_2 > qi_1$, so $p^* \notin P^*$.

If $l^* \subset V^*$, we can get

$$m(V^*) = i_0 + i_1 + i_2 + i_3 = i_0 + i_1 + \sum_{P\subset V^*}(m(P) - i_0 - i_1) \leqslant i_0 + i_1 + (q+1)i_2.$$

So $i_3 \leqslant qi_2$, which is contradictory to $i_3 > qi_2$, so $l^* \not\subset V^*$.

If $p^* \in V^*$, for $l^* \not\subset V^*$, we can get

$$m(V^*) = i_0 + i_1 + i_2 + i_3 = i_0 + \sum_{p^*\in l\subset V^*}(m(l) - i_0) \leqslant i_0 + (q^2+q+1)(i_1 - 1).$$

So $i_2 + i_3 \leqslant (q^2+q)i_1 - (q^2+q+1)$, which is contradictory to $i_2 + i_3 > (q^2+q)i_1$, so $p^* \notin V^*$.

So $p^* \notin P^*, l^* \not\subset V^*, p^* \notin V^*$.

$$m(V_4) = i_0 + i_1 + i_2 + i_3 + i_4 = i_0 + \sum_{p^*\in l\subset V_4}((m(l) - i_0) \leqslant i_0 + (q^3 + q^2 + q + 1)i_1.$$

We can then get $i_4 \leqslant (q^3 + q^2 + q)i_1 - i_2 - i_3$. So in order to keep i_4 as its upper bound, every line passing p^* is the heaviest line.

For $p^* \notin P^*$, the body generated by P^* and p^* is denoted by V, so we can get

$$m(P^*) \leqslant m(V) - i_0 = \sum_{p^*\in l\subset V}(m(l) - i_0) \leqslant (q^2+q+1)i_1$$

and $m(P^*) \leqslant (q^2+q+1)(i_0 - 1)$.

So $i_0 + i_1 + i_2 \leqslant (q^2+q+1)i_1$ and $i_0 + i_1 + i_2 \leqslant (q^2+q+1)(i_0 - 1)$.

We can also get

$$i_2 \leqslant (q^2+q)i_1 - i_0,$$
$$i_2 \leqslant (q^2+q)i_0 - i_1 - (q^2+q+1),$$

for $qi_1 < i_2 \leqslant (q^2+q+1)i_1 - i_0$, $i_1 > \dfrac{i_0}{q^2}$. For $p^* \notin V^*, l^* \not\subset V^*$, let x be the crossover point of V^* and l^*. As the value of every point in $V_4/\{V^*, l^*\}$ is declined to zero, the value of i_4 is the smallest, denoted by i_4^*, so we can get

$$i_0 + i_1 = m(x) + i_4^* \tag{2}$$

The number of planes containing the point x in V^* is (q^2+q+1). These planes cover every point in V^*/x $(q+1)$ times. Since the value of every plane is less than or equal to $i_0 + i_1 + i_2$, we can get

$$(q+1)m(V^*/x) + (q^2+q+1)m(x) \leqslant (q^2+q+1)(i_0 + i_1 + i_2).$$

That is

$$(q+1)[i_0 + i_1 + i_2 + i_3 - m(x)] + (q^2+q+1)m(x) \leqslant (q^2+q+1)(i_0 + i_1 + i_2).$$

So

$$q^2 m(x) + (q+1)i_3 \leqslant q^2(i_0 + i_1 + i_2) \tag{3}$$

Putting Formula (2) into Formula (3) we can get

$$q^2(i_0 + i_1 - i_4^*) + (q+1)i_3 \leqslant q^2(i_0 + i_1 + i_2).$$

So $i_4^* \geqslant \frac{q+1}{q^2}i_3 - i_2$. Some new necessary conditions have been gained.

The above necessary conditions are as follows:

(1) $\frac{i_0}{q^2} < i_1 \leqslant qi_0$;

(2) $qi_1 < i_2 \leqslant \min\left\{\frac{q^2}{q+1}(i_0+i_1), (q^2+q)i_1 - i_0, (q^2+q)i_0 - i_1 - (q^2+q+1)\right\}$;

(3) $qi_2 < i_3 \leqslant \frac{q^3}{q^2+q+1}(i_0+i_1+i_2)$;

(4) $\max\left\{i_0, i_1, \frac{q+1}{q^2}i_3 - i_2\right\} \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3$.

From (2), as $i_1 > qi_0 - q$, $\min\left\{\frac{q^2}{q+1}(i_0+i_1), (q^2+q)i_1 - i_0, (q^2+q)i_0 - i_1 - (q^2+q+1)\right\} = (q^2+q)i_0 - i_1 - (q^2+q+1)$.

For $qi_1 < i_2 \leqslant (q^2+q)i_0 - i_1 - (q^2+q+1)\}$, we can get $i_1 < qi_0 - q - \frac{1}{q+1}$. These two formulas are contradictory.

As $i_1 \leqslant qi_0 - q$, $\min\left\{\frac{q^2}{q+1}(i_0+i_1), (q^2+q)i_1 - i_0, (q^2+q)i_0 - i_1 - (q^2+q+1)\right\} = \min\left\{\frac{q^2}{q+1}(i_0+i_1), (q^2+q)i_1 - i_0\right\}$.

When $i_1 \geqslant \frac{i_0}{q}$, $\min\left\{\frac{q^2}{q+1}(i_0+i_1), (q^2+q)i_1 - i_0\right\} = \frac{q^2}{q+1}(i_0+i_1)$.

When $i_1 < \frac{i_0}{q}$, $\min\left\{\frac{q^2}{q+1}(i_0+i_1), (q^2+q)i_1 - i_0\right\} = (q^2+q)i_1 - i_0$.

From (2) and (3), we can get $\frac{q+1}{q^2}i_3 - i_2 > i_1$. So $\max\left\{i_0, i_1, \frac{q+1}{q^2}i_3 - i_2\right\} = \max\left\{i_0, \frac{q+1}{q^2}i_3 - i_2\right\}$.

From (4), we can get

$$i_0 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3,$$
$$\frac{q+1}{q^2}i_3 - i_2 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3,$$

so $i_3 \leqslant (q^3+q^2+q)i_1 - i_2 - i_0, i_3 \leqslant q^3 i_1$.

As $i_3 \leqslant \frac{q^2(i_0+i_2)}{q+1}$, $\max\left\{i_0, \frac{q+1}{q^2}i_3 - i_2\right\} = i_0$; as $i_3 \geqslant \frac{q^2(i_0+i_2)}{q+1}$, $\max\left\{i_0, \frac{q+1}{q^2}i_3 i_2\right\} = \frac{q+1}{q^2}i_3 - i_2$.

From these above conditions, the necessary conditions of type VI can be divided into six subclasses as follows.

Theorem 2 A difference sequence of type VI of q-ary linear codes with dimension 5 must satisfy one of the following six necessary conditions:

$$\text{VI-1:} \begin{cases} i_0 \leqslant i_1 \leqslant qi_0; \\ qi_1 < i_2 \leqslant \frac{q^2}{q+1}(i_0+i_1); \\ qi_2 < i_3 \leqslant \frac{q^3}{q^2+q+1}(i_0+i_1+i_2); \\ \frac{q+1}{q^2}i_3 - i_2 \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3. \end{cases}$$

$$\text{VI-2:} \begin{cases} \frac{i_0}{q} \leqslant i_1 \leqslant i_0; \\ qi_1 < i_2 \leqslant qi_0; \\ qi_2 < i_3 \leqslant \frac{q^2}{q+1}(i_0+i_2); \\ i_0 \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3. \end{cases}$$

$$\text{VI-3:} \begin{cases} \frac{i_0}{q} < i_1 \leqslant i_0; \\ qi_1 < i_2 \leqslant qi_0; \\ \frac{q^2}{q+1}(i_0+i_2) < i_3 \leqslant \frac{q^3}{q^2+q+1}(i_0+i_1+i_2); \\ \frac{q+1}{q^2}i_3 - i_2 \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3. \end{cases}$$

$$\text{VI-4:} \begin{cases} \frac{i_0}{q} < i_1 \leqslant i_0; \\ qi_0 < i_2 \leqslant \frac{q^2}{q+1}(i_0+i_1); \\ qi_2 < i_3 \leqslant \frac{q^3}{q^2+q+1}(i_0+i_1+i_2); \\ \frac{q+1}{q^2}i_3 - i_2 \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3. \end{cases}$$

$$\text{VI-5:} \begin{cases} \frac{i_0}{q^2} < i_1 \leqslant \frac{i_0}{q}; \\ qi_1 < i_2 \leqslant (q^2+q)i_1 - i_0; \\ qi_2 < i_3 \leqslant \frac{q^2}{q+1}(i_0+i_2); \\ i_0 \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3. \end{cases}$$

$$\text{VI-6:} \begin{cases} \frac{i_0}{q^2} < i_1 \leqslant \frac{i_0}{q}; \\ qi_1 < i_2 \leqslant (q^2+q)i_1 - i_0; \\ \frac{q^2}{q+1}(i_0+i_2) < i_3 \leqslant \frac{q^3}{q^2+q+1}(i_0+i_1+i_2); \\ \frac{q+1}{q^2}i_3 - i_2 \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3. \end{cases}$$

4 Sufficient Conditions of Type VI-1

In the following, we prove the sufficient conditions of type VI-1 in detail. First the boundary structure is proposed (taking the equal sign).

Lemma 1 The boundary sequence $(i_0, i_1, i_2, i_3, i_4)$ is the type VI-1 difference sequence as

$$i_1 = qi_0 - 2q - 1 \tag{4}$$

$$i_2 = qi_1 + q - 1 \tag{5}$$

$$i_3 = qi_2 + q \tag{6}$$

$$i_4 = (q^3 + q^2 + q)i_1 - i_2 - i_3 \tag{7}$$

where i_1 and i_4 are the upper bounds, and i_2 and i_3 are the lower bounds.

Proof From Formulae (4)-(7), we can get

$$m(l^*) = i_0 + i_1 = i_0 + q(i_0 - 2) - 1;$$

$$m(P^*) = i_0 + i_1 + i_2 = (q^2 + q + 1)(i_0 - 2);$$

$$m(V^*) = i_0 + i_1 + i_2 + i_3 = (q^3 + q^2 + q + 1)(i_0 - 2);$$

$$m(V_4) = i_0 + (q^4 + q^3 + q^2 + q + 1)(i_0 - 2) - (q^3 + q^2 + q + 1).$$

Let PG$(4, q)$ be a polyhedron whose vertexes are e_1, e_2, e_3, e_4, e_5 with dimension 4, as shown in Fig. 1. Let $\langle x_1, x_2, \cdots, x_t \rangle$ be a subspace generated by the points $x_1, x_2, \cdots, x_t$ with dimension $(t - 1)$.

In these lines passing by e_1, the first point from e_1 (not containing e_1) is denoted by $p_i\,(0 < i \leqslant q^3 + q^2 + q + 1)$.

We can construct the function $m(x)$ as follows:

$$m(x) = \begin{cases} i_0, & x = e_1; \\ i_0 - 3, & x = p_i \ (0 < i \leqslant q^3 + q^2 + q + 1); \\ i_0 - 2, & \text{others.} \end{cases}$$

So e_1 is the heaviest point, $\langle e_1, e_2 \rangle$ is the heaviest line (every line passing e_1 is the heaviest line), $\langle e_3, e_4, e_5 \rangle$ is the heaviest plane (every plane in $\langle e_2, e_3, e_4, e_5 \rangle$ is the heaviest plane), and $\langle e_2, e_3, e_4, e_5 \rangle$ is the heaviest body. So the boundary construction above satisfies both Formula (1) and the necessary condition of type VI, that is, $p^* \notin P^*$, $l^* \not\subset V^*$, $p^* \notin V^*$.

In order to get general constructions from the boundary construction, firstly we decrease i_1 to its lower bound by using the body set, while keeping the other i_j invariant (Lemma 2). Secondly, we increase i_2 to its upper bound, while keeping i_3 and i_4 invariant (Lemma 3). Thirdly, we increase i_3 to its upper bound, while keeping i_4 invariant (Lemma 4). Lastly, we decrease i_4 to its lower bound (Lemma 5).

Lemma 2 For any sequence $(i_0, i_1, i_2, i_3, i_4)$ satisfying Formulae (5)-(7), if it satisfies the formula as follows:

$$i_0 + h_1(q) \leqslant i_1 \leqslant qi_0 - 2q - 1: \tag{8}$$

then $(i_0, i_1, i_2, i_3, i_4)$ is the difference sequence of type VI-1, where $h_1(q) = q^6(q^3 - 1)$.

Proof We define some notations as follows (Fig. 2):

$$\langle e_3, e_4, e_5 \rangle = \{B_j | 0 < j \leqslant q^2 + q + 1\} = \{l_h | 0 < h \leqslant q^2 + q + 1\},$$

$$\langle e_2, e_3, e_4, e_5 \rangle / \langle e_3, e_4, e_5 \rangle = \{C_k | 0 < k \leqslant q^3\},$$

$$V_4 / \{e_1, \langle e_2, e_3, e_4, e_5 \rangle\} = \{A_i | 0 < i \leqslant q^4 - 1\}.$$

Define $m'(x)$ as follows:

$$m'(x) = \begin{cases} i_0, & x = e_1; \\ m(x) - 1, & x \in \langle A_i, C_k, l_h \rangle, \text{ and } A_i \notin \langle e_1, C_k, l_h \rangle; \\ m(x), & \text{others.} \end{cases}$$

The intersection of every body in the body set and every line in V_4 is at least one point. So in every change above, the value of every line decreases at least 1 ($\downarrow$ 1). The value of $\langle e_1, e_2 \rangle$ decreases 1, $\langle e_1, e_2 \rangle$ is also the heaviest line. The value of i_0 is invariant, and $i_1 \downarrow 1$. Body and plane meet in one line at least, so the value of every plane decreases $(q + 1)$ at least ($\downarrow (q + 1)$). The plane $\langle e_3, e_4, e_5 \rangle \downarrow (q + 1)$, and it is also the heaviest plane (P^*). Two bodies meet in one plane at least, so the value of every plane decreases $(q^2 + q + 1)$ at least.

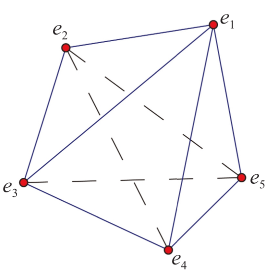

Fig. 1 Boundary structure model.

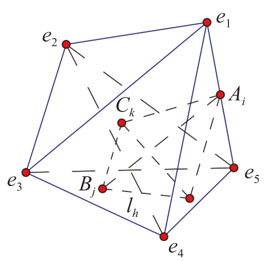

Fig. 2 i_1 decreasing model.

The body $\langle e_2, e_3, e_4, e_5\rangle \downarrow (q^2+q+1)$, and it is also the heaviest body (V^*).

$i_2 \downarrow q, i_3 \downarrow q^2, i_4 \downarrow q^3$, so they satisfy both Formulae (1) and (5)-(7).

The number of bodies $\langle A_i, C_k, l_h\rangle$ is $q^6(q^3-1)$. As we show all the bodies $\langle A_i, C_k, l_h\rangle$ satisfying the condition, we say i_1 decreases by one cycle. Once per cycle,

$$i_1 \downarrow q^6(q^3-1), C_k \downarrow q^5(q^3-1),$$
$$B_j \downarrow q^6(q^2-1), A_i \downarrow q^8,$$

where $A_i \in \langle e_1, e_3, e_4, e_5\rangle/\{e_1, \langle e_3, e_4, e_5\rangle\}$, otherwise point $A_i \downarrow q^5(q^3-1)$ (the number of such points is q^4-q^3). Obviously, the relation of p^*, l^*, P^*, and V^* does not change. The body $\langle e_2, e_3, e_4, e_5\rangle \downarrow (q^2+q+1)$ and it is also the heaviest body (V^*).

Assume that the cycles run at most ω_1 times. Since the value of every point is non-negative, we can get

$$i_0 - 3 - \omega_1 q^8 \geqslant 0, \text{ so } \omega_1 \leqslant \frac{i_0-3}{q^8}.$$

Let $\omega_1 = \left\lfloor \dfrac{i_0-3}{q^8} \right\rfloor$, so i_1 can decrease to

$$qi_0 - 2q - 1 - \omega_1 q^6(q^3-1) \geqslant$$
$$qi_0 - 2q - 1 - \frac{i_0-3}{q^8} q^6(q^3-1) \gg i_0.$$

If the request is $qi_0 - 2q - 1 - \omega_1 q^6(q^3-1) \geqslant i_0$, then $\omega_1 \leqslant \dfrac{(q-1)i_0-2q-1}{q^6(q^3-1)} < \dfrac{i_0-3}{q^8}$.

Let $\omega_1 = \left\lfloor \dfrac{(q-1)i_0-2q-1}{q^6(q^3-1)} \right\rfloor$, so i_1 can decrease to the value as follows:

$$qi_0 - 2q - 1 - \omega_1 q^6(q^3-1) < qi_0 - 2q - 1 -$$
$$\left[\frac{(q-1)i_0-2q-1}{q^6(q^3-1)} - 1\right] q^6(q^3-1) =$$
$$i_0 + h_1(q).$$

As $\omega_1 = \left\lfloor \dfrac{(q-1)i_0-2q-1}{q^6(q^3-1)} \right\rfloor$, i_1 can pass its lower bound, meanwhile the value of every point is non-negative, preparing for the increase of i_2 and i_3.

When the value of i_1 decreases, the value of B_j in $\langle e_3, e_4, e_5\rangle$ decreases less than the value of C_k. After one cycle, only $\langle e_3, e_4, e_5\rangle$ is the heaviest plane, and the value of B_j in $\langle e_3, e_4, e_5\rangle$ is $q^5(q^3-1) - q^6(q^2-1) = q^6 - q^5$ heavier than C_k, preparing for the increase of i_2 and i_3.

Lemma 3 For any sequence $(i_0, i_1, i_2, i_3, i_4)$ satisfying Formulae (6)-(8), if it satisfies the formula as follows:

$$qi_1 + q - 1 \leqslant i_2 \leqslant \frac{q^2}{q+1}(i_0+i_1) - h_2(q) \quad (9)$$

then $(i_0, i_1, i_2, i_3, i_4)$ is the difference sequence of type VI-1, where $h_2(q) = q^3(q^3-1)+q+1$.

Proof We define some notations as follows (Fig. 3):

$$\langle e_1, C_k\rangle/\{e_1, C_k\} = \{p(r,k) | 0 < r \leqslant q-1\},$$
$$\langle e_1, B_j\rangle/\{e_1, B_j\} = \{p(s,j) | 0 < s \leqslant q-1\}.$$

Define $m''(x)$ as follows:

$$m''(x) = \begin{cases} i_0, & x = e_1; \\ m'(x)+1, & x \in \langle B_j, C_k\rangle; \\ m'(x)-1, & x \in \langle B_j, p(r,k)\rangle/B_j, \text{ and} \\ & x = p(s,j) (0 < r \leqslant q-1); \\ m'(x), & \text{others.} \end{cases}$$

The number of lines $\langle B_j, C_k\rangle$ and $\langle B_j, p(r,k)\rangle$ is $q^3(q^3-1)$. Showing all the lines satisfying the condition, we say i_2 increases by one cycle. Once per cycle,

$$i_2 \uparrow q^3(q^3-1), C_k \uparrow q(q^3-1),$$
$$B_j \uparrow q^3(q-1), A_i \downarrow q^3,$$

where $A_i \in \langle e_1, e_3, e_4, e_5\rangle/\{e_1, \langle e_3, e_4, e_5\rangle\}$, otherwise point $A_i \downarrow (q^3+q^2+q)$ (the number of such an A_i is q^4-q^3).

Assume that the cycles run at most ω_2 times.

First, the value of every point is non-negative, so we can get

$$i_0 - 3 - \omega_1 q^8 - \omega_2 q^3 \geqslant 0, \text{ so } \omega_2 \leqslant \frac{i_0-3}{q^8} - \omega_1 q^5.$$

For $i_0 - 3 - \omega_1 q^5(q^3-1) - \omega_2(q^3+q^2+q) \geqslant 0$, then $\omega_2 \leqslant \dfrac{i_0-3}{q^3+q^2+q} - \omega_1 q^4(q-1)$.

Second, as $l^* \not\subset V^*$, we can get

$$(q+1)[i_0 - 2 - \omega_1 q^5(q^3-1) + \omega_2 q(q^3-1)] \leqslant$$
$$i_0 + qi_0 - 2q - 2 - \omega_1 q^6(q^3-1),$$

therefore $\omega_2 \leqslant \dfrac{q^4}{q+1}\omega_1$.

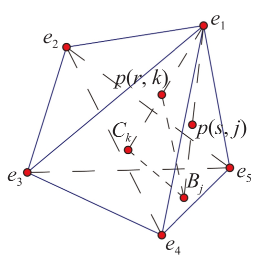

Fig. 3 i_2 **increasing model.**

For $i_1 = qi_0 - 2q - 1 - q^6(q^3-1)\omega_1$, we can get $\omega_1 = \dfrac{qi_0 - i_1 - 2q - 1}{q^6(q^3-1)}$, so $\dfrac{i_0 - 3}{q^3+q^2+q} - \omega_1 q^4(q-1) = \dfrac{qi_0 - 2q - 1 - i_1}{q^2(q+1)(q^3-1)}$ is the smallest of these two upper bounds.

Let $\omega_2 = \left\lfloor \dfrac{qi_0 - 2q - 1 - i_1}{q^2(q+1)(q^3-1)} \right\rfloor$, so i_2 can increase to

$$qi_1 + q - 1 + \omega_2 q^3(q^3-1) >$$
$$qi_1 + q - 1 + \left[\frac{qi_0 - 2q - 1 - i_1}{q^2(q+1)(q^3-1)} - 1\right] q^3(q^3-1) >$$
$$\frac{q^2}{q+1}(i_0 + i_1) - h_2(q),$$

so i_2 can increase near to its upper bound.

Therefore the above construction satisfies the necessary conditions, that is $p^* \notin P^*$, $l^* \not\subset V^*$, $p^* \notin V^*$.

The value of every point is more than 0, preparing for an increase of i_3 (Fig. 4).

When i_2 increases, the value of C_k increases more than the value of B_j. After part of the drops used as i_1 decreases, the value of the drop is $(q^3 - q)$, preparing for an increase of i_3.

Lemma 4 For any sequence $(i_0, i_1, i_2, i_3, i_4)$ satisfying Formulae (7)-(9), if it satisfies the formula as follows:

$$qi_2 + q \leqslant i_3 \leqslant \frac{q^3}{q^2+q+1}(i_0 + i_1 + i_2) - h_3(q) \quad (10)$$

then $(i_0, i_1, i_2, i_3, i_4)$ is the difference sequence of type VI-1, where $h_3(q) = q^3(q-1)$.

Proof Define $m'''(x)$ as follows:

$$m'''(x) = \begin{cases} i_0, & x = e_1; \\ m''(x) + 1, & x = C_k; \\ m''(x) - 1, & x = p(r,k); \\ m''(x), & \text{others}. \end{cases}$$

The number of points C_k and $p(r,k)$ is $q^3(q-1)$. As we show all the points satisfying the condition, we say i_3 increases by one cycle. Once per cycle, $i_3 \uparrow q^3(q-1), C_k \uparrow (q-1)$, every point in $V_4/\{e_1, V^*\}$ decreases by 1.

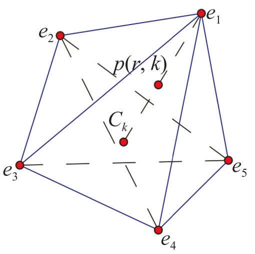

Fig. 4 i_3 **increasing model.**

Assume that the cycles run at most ω_3 times.

First, the value of every point is non-negative, so we can get

$$i_0 - 3 - \omega_1 q^8 - \omega_2 q^3 - \omega_3 \geqslant 0,$$

so $\omega_3 \leqslant i_0 - 3 - q^8\omega_1 - q^3\omega_2$.

For $i_0 - 3 - \omega_1 q^5(q^3-1) - \omega_2(q^3+q^2+q) - \omega_3 \geqslant 0$, then $\omega_3 \leqslant i_0 - 3 - q^5(q^3-1)\omega_1 - (q^3+q^2+q)\omega_2$.

Second, for $l^* \not\subset V^*$, we can get

$$(q+1)[i_0 - 2 - \omega_1 q^5(q^3-1) + \omega_2 q(q^3-1) +$$
$$(q-1)\omega_3] \leqslant i_0 + qi_0 - 2q - 2 - \omega_1 q^6(q^3-1),$$

so $\omega_3 \leqslant \dfrac{q^5(q^2+q+1)}{q+1}\omega_1 - q(q^2+q+1)\omega_2$.

Third, from $m(S) \leqslant m(S^*)$, we can get

$$(q+1)[i_0 - 2 - \omega_1 q^6(q^2-1) + \omega_2 q^3(q-1)] +$$
$$q^2[i_0 - 2 - q^5(q^3-1)\omega_1 + q(q^3-1)\omega_2 + (q-1)\omega_3] \leqslant$$
$$i_0 + (q+1)[qi_0 - 2q - 1 - \omega_1 q^6(q^3-1)] +$$
$$q - 1 + q^3(q^3-1)\omega_2,$$

so $\omega_3 \leqslant q^5\omega_1 - q(q+1)\omega_2$.

For $i_1 = qi_0 - 2q - 1 - q^6(q^3-1)\omega_1, i_2 = qi_1 + q - 1 + q^3(q^3-1)\omega_2$, we can get $\omega_1 = \dfrac{qi_0 - i_1 - 2q - 1}{q^6(q^3-1)}, \omega_2 = \dfrac{i_2 - qi_1 - q + 1}{q^3(q^3-1)}$.

So we can prove $q^5\omega_1 - q(q+1)\omega_2 = \dfrac{q^2(i_0+i_1) - (q+1)i_2 - q^2 - q - 1}{q^2(q^3-1)}$ is the smallest among the three upper bounds, while $p^* \notin P^*$, $l^* \not\subset V^*$, $p^* \notin V^*$.

Let $\omega_3 = \left\lfloor \dfrac{q^2(i_0+i_1) - (q+1)i_2 - q^2 - q - 1}{q^2(q^3-1)} \right\rfloor$, so i_3 can increase to

$$qi_2 + q + \omega_3 q^3(q-1) >$$
$$qi_2 + q + \left[\frac{q^2(i_0+i_1) - (q+1)i_2 - q^2 - q - 1}{q^2(q^3-1)(q^3-1)} - 1\right] \cdot$$
$$q^3(q-1) =$$
$$\frac{q^3}{q^2+q+1}(i_0 + i_1 + i_2) - h_3(q),$$

so i_3 can increase near to its upper bound.

Lemma 5 For any sequence $(i_0, i_1, i_2, i_3, i_4)$ satisfying Formulae (8)-(10), if it satisfies the formula as follows,

$$\frac{q+1}{q^2} i_3 - i_2 + h_4(q) \leqslant i_4 \leqslant (q^3+q^2+q)i_1 - i_2 - i_3 \quad (11)$$

then $(i_0, i_1, i_2, i_3, i_4)$ is the difference sequence of type VI-1, where $h_4(q) = q^3 - 1$.

Proof i_4 can decrease from its upper bound until the value of these points in $V_4/\{V^*, l^*\}$ is equal to 0. This moment i_4 can decrease to

$$i_0 + i_1 - m(e_2) =$$
$$i_1 + 2 + q^5(q^3-1)\omega_1 - q(q^3-1)\omega_2 - (q-1)\omega_3 =$$
$$i_0 + i_1 - \frac{i_3}{q^3} \qquad (12)$$

As i_4 decreases according to Formula (12), the method is as follows. In order to keep the values of $l^*, P^*,$ and V^* the same, we can get $i_4 \downarrow 1, e_2 \uparrow 1$, some $C_k \downarrow 1$ (where $C_k \in V^*/\{e_2, P^*\}$), and some point in $\langle e_1, e_2\rangle / \{e_1, e_2\} \downarrow 1$.

Define $m''''(x)$ as follows:

$$m''''(x) = \begin{cases} m'''(x) - 1, & x \in \langle e_1, e_2\rangle/\{e_1, e_2\}; \\ m'''(x) + 1, & x = e_2; \\ m'''(x) - 1, & x = C_k (C_k \neq e_2); \\ m'''(x), & \text{others.} \end{cases}$$

The number of points C_k is $q^3 - 1$. As we show all the points satisfying the condition, we say i_4 decreases by one cycle. Once per cycle, $i_4 \downarrow q^3 - 1, e_2 \uparrow q^3 - 1, C_k \in V^*/\{e_2, P^*\} \downarrow 1, p \downarrow (q^2 + q + 1)$ (where $p \in \langle e_1, e_2\rangle/\{e_1, e_2\}$).

Assume that the cycles run at most ω_4 times.

First, the value of C_k is non-negative, so we can get $i_0 - 2 - \omega_1 q^5(q^3-1) + \omega_2 q(q^3-1) + (q-1)\omega_3 - \omega_4 \geqslant 0$, so $\omega_4 \leqslant i_0 - 2 - \omega_1 q^5(q^3-1) + \omega_2 q(q^3-1) + (q-1)\omega_3$.

Second, for $m(l) < m(l^*)$ (where $l \subset V^*$ and $e_2 \in l$), we can get

$$m(l) = (q+1)(i_0 - 2) - q\omega_1 q^5(q^3-1) - \omega_1 q^6 \cdot$$
$$(q^2-1) + \omega_2 q^3(q-1) + \omega_2 q^2(q^3-1) + q(q-1)\omega_3 +$$
$$(q^3-1)\omega_4 - (q-1)\omega_4 \leqslant i_0 + i_1 - 1 =$$
$$i_0 + qi_0 - 2q - 2 - \omega_1 q^6(q^3-1),$$

so $\omega_4 \leqslant \omega_1 q^5 - \omega_2 q(q+1)$.

Third, for $m(P) \leqslant m(P^*)$ (where $P \subset V^*$ and $e_2 \in P$), we can get

$$m(P) = (q+1)[i_0 - 2 - \omega_1 q^6(q^2-1) +$$
$$\omega_2 q^3(q-1)] + q^2[i_0 - 2 - q^5(q^3-1)\omega_1 +$$
$$q(q^3-1)\omega_2 + (q-1)\omega_3] + (q^3-1)\omega_4 - (q^2-1)\omega_4 \leqslant$$
$$i_0 + i_1 + i_2 =$$
$$i_0 + i_1 + qi_1 + q - 1 + q^3(q^3-1)\omega_2,$$

so $\omega_4 \leqslant \omega_1 q^5 - \omega_2 q(q+1) - \omega_3$.

For $i_3 = qi_2 + q + q^3(q-1)$, we can get $\omega_3 = \dfrac{i_3 - qi_2 - q}{q^3(q-1)}$, and for $\omega_1 = \dfrac{qi_0 - i_1 - 2q - 1}{q^6(q^3-1)}, \omega_2 = \dfrac{i_2 - qi_1 - q + 1}{q^3(q^3-1)}$, we can prove $\omega_1 q^5 - \omega_2 q(q+1) - \omega_3 = \dfrac{q^3(i_0 + i_1 + i_2) - (q^2+q+1)i_3}{q^3(q^3-1)}$ is the smallest among the three upper bounds.

Let $\omega_4 = \left\lfloor \dfrac{q^3(i_0 + i_1 + i_2) - (q^2+q+1)i_3}{q^3(q^3-1)} \right\rfloor$, so i_4 can decrease to the value as follows:

$$i_0 + i_1 - \frac{i_3}{q^3} - (q^3-1)\omega_4 <$$
$$i_0 + i_1 - \frac{i_3}{q^3} - \left[\frac{q^3(i_0 + i_1 + i_2) - (q^2+q+1)i_3}{q^3(q^3-1)} - 1\right] \cdot$$
$$(q^3 - 1) =$$
$$\frac{q+1}{q^2} i_3 - i_2 + h_4(q),$$

so i_4 can decrease near to its lower bound.

From the above five lemmas, we can get Theorem 3.

Theorem 3 For q-ary linear codes with dimension 5, the sufficient condition that $(i_0, i_1, i_2, i_3, i_4)$ is the difference sequence of type VI-1 is that

(1) $h_0(q) \leqslant i_0$;

(2) $i_0 + h_1(q) \leqslant i_1 \leqslant qi_0 - 2q - 1$;

(3) $qi_1 + q - 1 \leqslant i_2 \leqslant \dfrac{q^2}{q+1}(i_0 + i_1) - h_2(q)$;

(4) $qi_2 + q \leqslant i_3 \leqslant \dfrac{q^3}{q^2+q+1}(i_0 + i_1 + i_2) - h_3(q)$;

(5) $\dfrac{q+1}{q^2} i_3 - i_2 + h_4(q) \leqslant i_4 \leqslant (q^3 + q^2 + q)i_1 - i_2 - i_3$,

where $h_0(q) = q^6(q^2+q+1) + 2 + \dfrac{3}{q-1}$, $h_1(q) = q^6(q^3-1), h_2(q) = q^3(q^3-1) + q + 1$, $h_3(q) = q^3(q-1), h_4(q) = q^3 - 1$.

Therefore, Theorem 3 is proved completely.

Let $N(i)$ be the number of the difference sequences of the VI-1 class with $i_0 \leqslant i$, and $M(i)$ the number of the sequences satisfying the necessary condition in Theorem 3 with $i_0 \leqslant i$. By calculating we can get $\lim\limits_{i \to +\infty} \dfrac{N(i)}{M(i)} = 1$.

5 Conclusions

We researched the type VI weight hierarchy of q-ary linear codes with dimension 5 and found some new necessary conditions of type VI. We then divided the type VI difference sequence into six classes. By the finite projective geometry method, we researched the type VI-1 weight hierarchy and got almost all possible type VI-1 weight hierarchies.

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