Differential Coherent Algorithm Based on Fast Navigation-Bit Correction for Airborne GNSS-R Software Receivers

Yiran Wang  
School of Electronic Information Engineering, Beihang University, Beijing 100191, China

Bo Zhang  
School of Electronic Information Engineering, Beihang University, Beijing 100191, China

Dingrong Shao  
School of Electronic Information Engineering, Beihang University, Beijing 100191, China

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Differential Coherent Algorithm Based on Fast Navigation-Bit Correction for Airborne GNSS-R Software Receivers

Yiran Wang, Bo Zhang*, and Dingrong Shao

Abstract: Signals from the Global Navigation Satellite System (GNSS) scatter over the sea surface resulting in relatively low Signal-to-Noise Ratios (SNR). A differential coherent algorithm is given here to improve the SNR and reduce the performance degradation due to the Squaring-Loss and the navigation-bit effect. The algorithm uses fast navigation-bit correction for Delay-Doppler Maps (DDM) in airborne Global Navigation Satellite Signal Reflectometry (GNSS-R) software receivers. The system model is introduced with an analysis of the statistical properties with simulations to support the theoretical analysis. Field experiments with real airborne receivers then demonstrate the effectiveness of this algorithm. Comparisons with test results show that this algorithm offers a significant SNR gain over conventional algorithms.

Key words: satellite navigation; signal processing; Global Navigation Satellite Signal Reflectometry (GNSS-R); Delay-Doppler maps; differential coherence; signal-to-noise ratio

1 Introduction

Global Navigation Satellite Signal Reflectometry (GNSS-R), originally proposed by the European Space Agency in 1993[1], has great potential for remote sensing of geophysical parameters such as soil moisture[2, 3], sea ice concentrations[4], ocean altimetry[1], and sea state retrieval[5-9].

In GNSS-R mesoscale altimetry, a receiver collects both the right-hand circular polarization signals directly emitted by navigation satellites and the forward scattered left-hand elliptical polarization signals reflected from the sea surface, which have high variability since the scatter distribution is closely related to the sea state and the sea-surface roughness. The altimetry information is found from studying the Delay-Doppler Maps (DDM)[10-12] from the software receiver, which gives the relative delay estimation between the direct and reflected signals[13-15]. The DDM processor correlates the received signals with a local replica of the transmitted navigation signals[1, 6, 9]. However, the Signal-to-Noise Ratio (SNR) of the GNSS signal scattered on the sea surface is relatively poor, which will reduce the precision and cause serious altimetry errors, so a longtime integration is required.

The two main integration types are coherent and non-coherent integrations. Coherent integration does not improve the overall SNR in a dynamic receiver, such as space-borne or airborne receivers, because the total correlation power is reduced by Doppler filtering effects due to the phase information preserved in the integration process. For low dynamics receivers, such as the ground-based receivers, the Doppler effects can be treated as constant within the glistening zone[16], so the coherent integration algorithm can be used and the output SNR is enhanced by a factor equal to the integration time. The maximum SNR improvement of the coherent algorithm is then subject to the longest integration time, which is limited by the navigation-bit effect (i.e., the phase jumps caused by the navigation-bit polarity inversion). Recent progress has been made towards correcting the navigation-bit effect
with demodulated baseband data\cite{17}. However, these methods require considerable memory resources and degrade the real-time receiver performance. Therefore, the majority of GNSS-R DDMs use non-coherent integration to avoid these disadvantages of coherent systems, yet their results are still unsatisfactory because of serious SNR degradation due to the Squaring-Loss\cite{18,19}.

The SNR performance is enhanced here to improve the accuracy of relative delay estimates by a differential coherent integration algorithm using fast navigation-bit correction. Differential coherent integration, which multiplies the current coherently integrated pre-detection samples with the conjugate complex of the previous samples, has been proposed for spread spectrum communication systems\cite{20} and for high sensitivity navigation receivers\cite{21}. The key idea is that given that the noise is theoretically uncorrelated in time with itself, better detect performance can be achieved by effective elimination of the Squaring-Loss. Differential coherent algorithms are less sensitive to navigation-bit effects than coherent methods\cite{22,23}, so the SNR degradation caused by bit inversion can be simply reduced by first correcting the navigation-bit effect using polarity decisions based on the DDM output peak values of the direct signals and then computing the accumulation results of the differential coherent integration.

2 Signal and System Model

The GNSS-R observation is usually in the form of the DDM obtained from processing the GPS C/A signals scattered from the sea surface as illustrated in Fig. 1. To relate these observations with the correlation time of the sea surface under observation using Kirchhoff approximation\cite{24,25}, the DDM in one pre-detection integration period $T_i$ needs to be expressed as a function of the sea surface characteristics as\cite{26}

$$ Y(t_0, \tau, f_D) = T_i \int \int D(r)d(t_0)A(\Delta \tau; t_0) \times S(\Delta f)g(r, t_0 + \tau)d^2r $$

(1)

where $D$ is the antenna radiation pattern in terms of the complex amplitudes, $d(t)$ denotes the baseband data randomly overlapped with a period of 20 ms and $\Delta \tau = \tau - (R_t + R_s)/c$ where $\tau$ is the time delay, and $\Delta f$ is the frequency deviation after compensating for the Doppler shift. The triangle function $A$ is defined as $A(\Delta \tau) = 1 - |\Delta \tau|/\tau_c$ if $|\Delta \tau| \leqslant 1 + \tau_c/T_i$ and is approximately zero elsewhere (the generic value of $\tau_c$ is the period of one chip of the transmitting C/A code, i.e., 1/1023 ms). $S(\Delta f)$ is given by

$$ S(\Delta f) = \sin c(\pi \Delta f T_i) \exp(-\pi \alpha \Delta f T_i) $$

where

$$ \sin c(\pi \Delta f T_i) = \sin(\pi \Delta f T_i)/\pi \Delta f T_i $$

The complex function $g$ describes the scattering geometry and the scattering surface,

$$ g(r, t + \Delta \tau) = -\mathcal{M}\exp[-2\pi i(f_{IF} + f_D(r, t))\Delta t] \times \exp[i K(R_t + R_s)q^2/q_z] $$

(2)

where $\mathcal{M}$ is the Fresnel reflection coefficient at a given polarization, $f_{IF}$ is the mid-frequency of the transmitted signal which shifted by a Doppler frequency $f_D$, $K = 2\pi f_{IF}/c$, and the scattering vector $q$ is defined as $q = (q_x, q_z) = K(R/R_t - T/R_t)$ = $K(n - m)$ with $n$ and $m$ as the unit vectors in the incident and scattering directions.

Using the bistatic radar observations given in Appendix A, the power distribution after differential correlation can be expressed as a 2-D convolution:

$$ \langle Y_k(\tau, f_D) \cdot Y_{k-1}^*(\tau, f_D) \rangle = d_k d_{k-1} A^2(\Delta \tau) |\sin c(2\pi \Delta f T_i)|^2 * * T_i^2 D^2(R(\tau, f_D))\sigma_0(R(\tau, f_D)) \langle J(\tau, f_D) \rangle $$

(3)

The final output DDM of the GNSS-R receiver using the differential coherent algorithm is then the average power as a function of the delay $\tau$ obtained by multiplying the current integration result with the conjugated complex of the previous result and then accumulating over the differential integration time $T_k = (M - 1) \times T_i$ as

$$ | \sum_{k=2}^{M} \langle Y_k(\tau, f_D) \cdot Y_{k-1}^*(\tau, f_D) \rangle | = $
As with the traditional coherent algorithms, the actual processing of the GPS signals has a pre-detection integration time of 1 ms (i.e., one C/A code period) and a 50% chance of baseband bit inversion with a period of 20 ms. Thus, the usable $T_k$ in the differential coherent algorithm is limited by this navigation-bit effect, which determines the output SNR improvement that can be achieved.

This navigation-bit effect is eliminated here by a differential coherent algorithm using a fast navigation-bit correction as shown in Fig. 2. This scenario is possible since for a general airborne GNSS-R receiver, the peak value polarities of the DDMs obtained by differential correlation of the pre-detection integration results, can be observed in the direct channel due to the much better SNR environment. With this scenario, the baseband bit inversions of the transmitted signal can be detected and compensated for by polarity decisions using the peak values. Thus, as long as the baseband data varies from one DDM to the next computed each millisecond, the polarity is negative; otherwise it is positive. Moreover, since the relative delay between the direct signal and the corresponding reflected signal is much smaller than one C/A code period, the bit inversion can be assumed to occur almost instantaneously in the reflected channel.

Consequently, the principal advantage of the present technique is that it significantly reduces the memory requirement and the extreme degradation of the realtime performance of the conventional scheme by eliminating the navigation-bit effect with demodulated baseband data.

3 Performance Analysis

The SNR performance can be improved by the differential coherent algorithms since the noise samples are independent of each other and their accumulation leads to a noise term equivalent to an asymptotically null power. This section theoretically characterizes the differential algorithm performance based on the detection and false alarms probabilities.

3.1 Pre-detection coherent integration

To simplify the analysis, the scattered field at the receiver position is expressed as\cite{27}

\[
U(R, t) = \int W(r, t)d(t - \tau_r(r, t))c(t - \tau_r(r, t))\times 
\exp[-2\pi i(f_r + f_D(r, t))(t - \tau_r(r, t))]dr
\]  

(5)

where $c(t)$ denotes the received spreading code; $W(r, t)$ represents the complex amplitude factors including all the radar equation parameters such as the voltage antenna pattern of the transmitting satellite, the path loss, and the complex reflectivity of the scattering ocean surface; and $\tau_r(r, t)$ is the time delay of the

![Fig. 2 Schematic diagram of a GNSS-R software receiver using the differential coherent algorithm.](image-url)
general reflected signal impinging the sea surface at position \( r \) received at the down-looking antenna. Then, the total received and down-converted scattered GNSS signal can be simplified to a coherent summation of the scattered signal and the thermal noise in the generic down-looking antenna beam,

\[
r(t) = U(R, t) \exp[i(\varphi_{LO} + 2\pi f_{LO}t)] + n_R(t)
\]

where \( \varphi_{LO} \) and \( f_{LO} \) are the phase and frequency of the local oscillator. The variance of the complex, zero-mean, additive white Gaussian noise \( n_R(t) \) in the reflected channel is \( \sigma_n^2 = 2k_B T_R BF \) with \( F \) is the noise figure of the receiver, \( B \) is the pass-bandwidth, \( T_R \) is the noise temperature of \( n_R(t) \), and \( k_0 \) is the Boltzmann constant.

Processing the sampled and down-converted reflected signal with PRN reference code \( c_i \) and pre-detection integration of \( N = T_i/T_s \) chips with \( T_i \) as the pre-detection integration time and \( T_s \) as the sample period gives the DDM result after integration as

\[
S_k = \int W_{r, t} d_k \lambda_k (\Delta t_k) \sin c(\pi \Delta f_k T_i) \times \exp(\pi i \Delta f_k T_i + \varphi_k) dt + \omega_k
\]

where \( \Delta f_k \) and \( \varphi_k \) are the average frequency deviation and phase mismatch during one pre-detection integration period and \( \omega_k = \omega_{k, I} + i\omega_{k, Q} \) denotes the zero-mean, complex-valued, Gaussian distribution with variance \( \sigma_\omega^2 = 2k_0 T_R BFN \).

### 3.2 Differential correlation

If the consecutive product terms of the integrator are independent of each other, the reconstruction formula corresponding to Eq. (4) has the form

\[
\Psi_{\text{diff}} = |\sum_{k=2}^{M} \text{Re}(S_k S_{k-1}^*)| =
\]

\[
|\sum_{k=2}^{M} \text{Re}((S_{k, I} + jS_{k, Q})(S_{k-1, I} - jS_{k-1, Q}))| =
\]

\[
|\sum_{k=2}^{M} (S_{k, I} S_{k-1, I} + S_{k, Q} S_{k-1, Q})| =
\]

\[
|\sum_{k=2}^{M} ((\Omega_I)^2 + (\Omega_Q)^2 - (\mathcal{E}_I)^2 + (\mathcal{E}_Q)^2)|
\]

As can be seen in Eq. (8), as with the traditional differentially coherent integration schemes\cite{28}, only the real part of the product \( S_k S_{k-1}^* \) has been included for simplicity. The analysis could also use both the real and imaginary parts but that would not affect the results.

Usually, the presence of scattered signal is declared when \( \Psi_{\text{diff}} \) with fixed values of \( r \) and \( f_D \) exceeds a decision threshold \( T_h \) and the signal is correctly synchronized. Then, the relative delay estimates between the desired cross-correlation peaks of the direct and reflected signals are presented as the altimetry information. However, if there is no scattered signal present or if it is not synchronized, a threshold crossing causes a false alarm which will result in serious altimetry errors. The analytical derivation uses the following two conditions:

**Hypothesis 0** The signal in the reflected channel is absent or has misaligned code.

**Hypothesis 1** The signal in the reflected channel is present and is correctly synchronized with the replica; thus, the desired cross-correlation peak can be identified.

Applying the properties of Gaussian random variables\cite{29,30}, with Hypothesis 0, \( \Psi_{\text{diff}} \) can be treated as the difference between two independent central \( \chi^2 \) distributions with \( 2(M - 1) \) degrees of freedom because of the orthogonal properties of the transmitted C/A code. With the condition in Hypothesis 1, \( \Psi_{\text{diff}} \) can be deemed to be a non-central \( \chi^2 \) distribution minus a central \( \chi^2 \) distribution, both with \( 2(M - 1) \) degrees of freedom.

### 3.2.1 Analysis of Hypothesis 1

Following the previous derivations, since the input signals of the differential coherent integrator in every period of the pre-detection coherent integration are independent, \( \Psi_{\text{diff}} \) can be expressed as

\[
\Psi_{\text{diff}} = \Psi_{\text{non-c}} - \Psi_c =
\]

\[
\frac{\sigma_w^2}{2} \sum_{k=2}^{M} ((\Omega_I)^2 + (\Omega_Q)^2) - \frac{\sigma_w^2}{2} \sum_{k=2}^{M} ((\mathcal{E}_I)^2 + (\mathcal{E}_Q)^2)
\]

\[
(10)
\]

When the received signal is aligned with the local replica, the summation of the non-central \( \chi^2 \) components in Eq. (10) is \( \Psi_{\text{non-c}} = \Psi_{\text{non-c,I}} + \Psi_{\text{non-c,Q}} \), which is a non-central \( \chi^2 \) distribution with \( 2(M - 1) \) degrees of freedom and the non-centrality parameter,

\[
\lambda_{\text{non-c}} = \lambda_{\text{non-c,I}} + \lambda_{\text{non-c,Q}} = 2(M - 1)
\]

\[
\frac{1}{\sigma_w^2}
\]

\[
(11)
\]
Thus $\Psi_c$ corresponds to a central $\chi^2$ distribution with $2(M - 1)$ degrees of freedom. The Probability Density Functions (PDF) of $\Psi_{\text{non-c}}$ and $\Psi_c$ are\cite{29}

$$f_{\Psi_{\text{non-c}}} (\Psi_{\text{non-c}}) = \exp \left( -\frac{\Psi_{\text{non-c}} + \lambda_{\text{non-c}}}{2} \right) \times \left( \frac{\Psi_{\text{non-c}}}{\lambda_{\text{non-c}}} \right)^{M-2} I_{M-2} \left( \sqrt{\Psi_{\text{non-c}} \lambda_{\text{non-c}}} \right)$$

(12)

$$f_{\Psi_c} (\Psi_c) = \frac{1}{\sigma_c^2 \Gamma (M - 1)} \left( \frac{\Psi_c}{\sigma_c^2} \right)^{M-2} \exp \left( -\frac{\Psi_c}{\sigma_c^2} \right)$$

(13)

where $I_{M-2}(\cdot)$ is the modified Bessel function of the first kind of order $M - 2$. The detection probability is then expressed as

$$P_d = p((\Psi_{\text{non-c}} - \Psi_c) > T_h) = 1 - p \left[ \frac{\Psi_{\text{non-c}}}{\Psi_c + T_h} < 1 \right]$$

(14)

Expanding $\Phi$ as the summation of $\Psi_c$ and $T_h$, then $\Phi$ can be seen to be the distribution $\Psi_c$ shifted by $T_h$ as $f_{\Phi}(\Phi) = f_{\Psi_c}(\Phi - T_h)$. Since $\Psi_{\text{non-c}}$ and $\Phi$ are independent of each other and the PDF of $\Phi$ is not smaller than $T_h$, which is always positive, the detection probability can be further expressed as:

$$P_{\text{diff}} (\Psi_{\text{diff}}) = 1 - p \left( \frac{\Psi_{\text{non-c}}}{\Psi_c + T_h} < 1 \right) = 1 - \int_{T_h}^{\infty} f_{\Psi_{\text{non-c}}} (\Psi_{\text{non-c}}) \times f_{\Psi_c} (\Psi_c - T_h) d\Psi_{\text{non-c}} d\Phi$$

(15)

Inserting Eqs. (12) and (13) into Eq. (15) gives $P_d$.

### 3.2.2 Analysis of Hypothesis 0

With Hypothesis 0, applying the modulus transformation to the difference of the non-central and central random variables with $2(M - 1)$ degrees of freedom\cite{30} gives the PDF,

$$f_{\Psi_{\text{diff}}} (\Psi) = \frac{2}{(M - 2)! \sigma_{\text{diff}}^2} \exp \left( -\frac{\Psi}{\sigma_{\text{diff}}^2} \right) \times \sum_{i=0}^{M-2} \left( \frac{\Psi}{\sigma_{\text{diff}}^2} \right)^i (2(M - 2) - i)! (M - 2 - i)!$$

(16)

Then, the probability of a false alarm, $P_{fa}$, is obtained by integrating Eq. (16) by parts:

$$P_{fa} (\Psi_{\text{diff}}) = \frac{2}{(M - 2)! \sigma_{\text{diff}}^2} \exp \left( -\frac{\Psi_{\text{diff}}}{\sigma_{\text{diff}}^2} \right) \times \sum_{i=0}^{M-2} \sum_{j=0}^{i-2} \left( \frac{\Psi_{\text{diff}}}{\sigma_{\text{diff}}^2} \right)^{M-i-2} (2(M - 2) - i)! (M - i - 2)! (i - j)!$$

(17)

Equation (17) is the best to our knowledge and embodies one of the innovative contributions of this paper.

### 3.3 Simulations

The present algorithm is compared here to the conventional non-coherent algorithm\cite{31}

$$\Psi_{\text{con}} = \sum_{k=1}^{M} S_k^2$$

(18)

The properties of non-central and central $\chi^2$ random variables with $2M$ degrees of freedom\cite{30,32} can be used to find the false alarm and detection probabilities for the conventional non-coherent algorithm as

$$P_{fa} (\Psi_{\text{con}}) = \exp \left( -\frac{\Psi_{\text{con}}}{\sigma_{\text{con}}^2} \right) \sum_{k=0}^{M-1} \frac{1}{k!} (\frac{\Psi_{\text{con}}}{\sigma_{\text{con}}^2})^k$$

(19)

$$P_d (\Psi_{\text{con}}) = Q_M \left( \frac{1}{\sigma_{\text{con}}} \frac{\Psi_{\text{con}}^2}{\sigma_{\text{con}}^2} \right)$$

(20)

where $Q_M(x, y)$ is the generalized Marcum $Q$-function of order $M$ and the non-centrality parameter $\lambda_{\text{con}}$ is equal to $\lambda_{\text{con}} \approx M \cdot \left( \int_{W_{r,1}} W_r \, dr \right)^2$.

The algorithms are compared based on the detection probability versus the Carrier-to-Noise Ratio (CNR) and the receiver operating characteristics for the detection probability versus the false alarm probability. The analysis assumes that only GPS C/A codes with a code length of 1023 and signal powers of $-153 \text{ dBW}$ to $-158.5 \text{ dBW}$ are received by the GNSS-R receiver antennas. The effect of the Radio Frequency (RF) front-end filter is neglected and the received signal is treated as an ideal BPSK modulation. The campaign results from different scenarios are evaluated by means of Monte Carlo simulations after introducing the theoretical background of the differential algorithm and giving the analytical simulation conditions.

The $P_d$ of the DDM of the GNSS-R acquisition processing is simply related to the CNRs of the reflected signal (CNR). A plot of $P_d$ versus the CNR for different values of $M$ without consideration of the Doppler effect is given in Fig. 3 assuming that the fixed probability of false alarms $P_{fa}=1 \times 10^{-7}$. The results show that the present algorithm outperforms the conventional one with a higher detection probability for a given CNR. Figure 4 further compares for $f_d=0$ Hz versus $f_d=200$ Hz with a generic $T_i$ of 1 ms. The simulations show that the sensitivity degrades rapidly when the Doppler effect is taken into account; thus, a long $T_i$ or a large $M$ is inevitable.

The receiver operating characteristics are compared for $\text{CNR}=25$, 29, and 33 dB-Hz without frequency
deviations in Fig. 5. The operating characteristics for $\text{CNR}_R=30 \, \text{dB-Hz}$ with a serious $f_d$ of 300 Hz are shown in Fig. 6 where the present differential coherent algorithm gives better detection performance than the conventional algorithm for the simulation conditions.

The simulations clearly show that the differential coherent integration algorithm provides a considerable gain in $P_d$, compared with the most common non-coherent algorithm, with the specific gain depending on the simulation scenario and parameters. Also, there are no obvious differences in the Doppler effects between these two algorithms. Thus, the non-coherent algorithm always gives worse performance due to the fact that the Squaring-Loss effect is inevitable.

4 Field Experiments

To further illustrate the effectiveness of the current algorithm, field experiments were conducted with airborne receivers.

The raw data set (i.e., the reflected and direct signals together with the Y-7 aircraft kinematic data) was gathered during a flight from 7:50 am to 11:00 am as the aircraft overflew the South China Sea off the coast of Boao (Hainan, China) for approximately 45 km along a 220 km route at a speed of about 450 km/h. The altitude of the aircraft during the measurements was almost constant at approximately 5200 m while the aircraft twice overflew the measurement zone in the northeast-southwest direction as shown in Fig. 7a. This analysis focuses on the data processing of the 30 minute long

Fig. 3 $P_d$ for the differential coherent strategy and the conventional non-coherent strategy with $P_{th} = 1 \times 10^{-7}$, $F = 3 \, \text{dB}$, $f_d = 0 \, \text{Hz}$, and $T_i = 1 \, \text{ms}$.

Fig. 4 $P_d$ for the differential coherent strategy and the conventional non-coherent strategy with $P_{th} = 1 \times 10^{-7}$, $F = 3 \, \text{dB}$, and $T_i = 1 \, \text{ms}$.

Fig. 5 Receiver operating characteristics for the differential coherent algorithm and the conventional algorithm with $T_i = 1 \, \text{ms}$, $M = 20$, and $f_d = 0 \, \text{Hz}$.

Fig. 6 Receiver operating characteristics for the differential coherent algorithm and the conventional algorithm with $T_i = 1 \, \text{ms}$ and $M = 20$.

Fig. 7 Remote sensing information in the flight zone: (a) total flight trajectory; (b) elevation and azimuth of the visible satellites.
track beginning at 9:40 am. During the tracking, PRNs 02, 04, 17, 27, and 28 were visible with elevation angles between 17° and 66° as seen in the sky plot in Fig. 7b and in Table 1. The elevation angles of PRNs 02 and 27 were too low for use in the altimetry processing, so only PRNs 04, 17, and 28 with reasonable elevation angles were used.

The direct signals were received by an ordinary Right Hand Circularly Polarized (RHCP) antenna while the weak scattered signals collected by a 12 dB gain Left Hand Circularly Polarized (LHCP) antenna having a four-subunit antenna array. The raw signals in the direct and reflected channels were acquired by an eight-channel receiver sampled at 20.4444 MHz, 2 bit quantized and preprocessed with a dedicated software having one subunit for each signal. Cross-correlations were computed at 201 delay lags while the Doppler dimension spanned from −2000 Hz to 2000 Hz with steps of 50 Hz. Standard algorithms were used for the preprocessing of the direct signal, while the reflected signal was analyzed using the initial settings with the frequency and time delay information derived from those of the direct unit.

In the present differential algorithm, the navigation-bit effects were eliminated using the conventional initial parameters plus the phase inversion of the navigation bit introduced as an important initial parameter obtained from the polarity decision of the differential coherent peak of the direct unit which uses the standard differential algorithm as in Fig. 2. The successful detection of the bit inversion is an essential step for the GNSS-R processing. The observed peak values of $Y_{D,k}Y_{D,k-1}^*$ and $Y_kY_{k-1}^*$ derived in Fig. 2 were calculated using PRNs 04, 17, and 28 for continuous 200 ms periods, they are given in Figs. 8-10 which show that for all three PRNs, the peak polarity of $Y_{D,k}Y_{D,k-1}^*$ becomes negative five times with intervals of integer multiples of 20 ms, corresponding to random baseband bit inversions with a period of 20 ms as analyzed previously. The bit inversions can not be directly determined from the reflected signal because of the poor SNR.

Since the performance of the present algorithm can not be determined precisely without knowing the accurate location of the bit phase inversion, the effectiveness of the present algorithm is tested by comparing the non-coherent algorithm (NON), the differential coherent algorithm without data bit correction (DIFF-bit), and the scenario

$$\psi_{\text{diff-abs}} = \sum_{k=2}^{M} |\text{Re}(S_k S_{k-1}^*)| \quad (21)$$

which uses the absolute value to remove the dependence of the product of the navigation message (DIFF-abs). These are compared with various non-coherent/differential coherent accumulation times with the results summarized in Table 2.

The results in Table 2 show that the output SNR performance of the differential scenarios improves significantly in contrast with the non-coherent scenario because of the elimination of the Squaring-Loss. Thus, the present differential method based on fast bit correction outperforms both the differential method without bit correction and outperforms the method in Eq. (21) because the Squaring-Loss can not be completely avoided by using the absolute value. Furthermore, while no data bit inversions occurred during integral times such as $M = 20$ in Table 2, the output SNRs of the two purely differential methods with small squaring-losses are identical on all PRNs; thereby, conforming the analysis in Figs. 8-10.

The results show good agreement with the derived models despite the fact that the output SNRs are not enhanced by increasing the integral time with PRN 04.

<table>
<thead>
<tr>
<th>Table 1 Elevation information of the visible satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation angle (°)</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>PRN02</td>
</tr>
<tr>
<td>PRN04</td>
</tr>
<tr>
<td>PRN17</td>
</tr>
<tr>
<td>PRN27</td>
</tr>
<tr>
<td>PRN28</td>
</tr>
</tbody>
</table>

Fig. 8 Peak values of $Y_{D,k}Y_{D,k-1}^*$ and $Y_kY_{k-1}^*$ using PRN 04.
Table 2 Comparison of the test results.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$M$</th>
<th>Present DIFF</th>
<th>DIFF-bit</th>
<th>DIFF-abs</th>
<th>NON</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak ($\times 10^7$)</td>
<td>Mean ($\times 10^5$)</td>
<td>SNR (dB)</td>
<td>Peak ($\times 10^7$)</td>
<td>Mean ($\times 10^5$)</td>
</tr>
<tr>
<td>PRN 04</td>
<td>20</td>
<td>3.0224</td>
<td>5.7235</td>
<td>17.227</td>
<td>3.0224</td>
</tr>
<tr>
<td>PRN 17</td>
<td>20</td>
<td>1.7879</td>
<td>5.0312</td>
<td>15.507</td>
<td>1.7879</td>
</tr>
<tr>
<td>PRN 28</td>
<td>20</td>
<td>4.7331</td>
<td>5.0892</td>
<td>19.685</td>
<td>4.7331</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>32.740</td>
<td>15.469</td>
<td>23.256</td>
<td>28.723</td>
</tr>
</tbody>
</table>

5 Conclusions

This paper presents a differential coherent algorithm based on fast navigation-bit correction that overcomes the degradation of the SNR caused by the Squaring-Loss and the navigation-bit effect in GNSS-R processing. Statistical analyses and Monte Carlo simulations show that the present algorithm is more effective than the conventional non-coherent algorithm. Field tests with a real airborne receiver are also used to compare this method with three other common integration methods. The results show that the current technique significantly outperforms the traditional methods. The differential coherent algorithm effectively eliminates the Squaring-Loss which improves the SNR performance. Moreover, decomposition of the continuous direct signals to the scale of the coherent integration period and comparisons of the peak polarity after differential cross-correlations can locate navigation bit inversions in the code phase measurements. These singularities can be analyzed to correct the navigation-bit effect in the reflected signal processing with no need to wait for time consuming synchronization and demodulation.

Appendix A

The calculation of $\langle Y_k(\tau, f_D) \cdot Y_{k-1}^*(\tau, f_D) \rangle$ requires additional simplifications for airborne receivers with speeds of 400-600 km/h since the distance changes between the transmitter, the receiver, and the reflection point are practically negligible within the integration range.
period (typically 1 ms). In addition, the slowly changing functions \( R \) and \( D \) can be treated as constants, with only the large-scale surface elevation, \( \zeta \), significantly contributing to the statistics of the time-delayed average power after the differential coherent integration. From Eq. (1) and assuming that the rough sea-surface within some fixed area can be represented as a single-valued function \( \zeta(r, t) \) of the plane vector \( r = (x, y, 0) \), the power distribution can be expressed as:

\[
\begin{align*}
Y_k(t, f_D) \cdot Y_{k-1}(t, f_D) &= T_i^2 \int \int \left[ (D_k d_k A_k S_k g_k) \times \right. \\
& \left. (D_{k-1} d_{k-1} A_{k-1} S_{k-1} g_{k-1}) \right] dr_k dr_{k-1} = \\
& T_i^2 \frac{16\pi}{2} \int \int \frac{D_k d_k A_k S_k g_k}{R_k R_{k-1}} \times \\
& \frac{D_{k-1} d_{k-1} A_{k-1} S_{k-1} g_{k-1}}{R_{k-1} R_{k-1}} \cdot \frac{q_k^2 q_{k-1}^2}{q_z} \times \\
& \exp[iK(R_{k} + R_{k-1} - R_{k-1} - R_{k-1})] \times \\
& \Phi(r_k, r_{k-1}) dr_k dr_{k-1} \\
& \text{(A1)}
\end{align*}
\]

where \( R_{k} = |T_k - r_k| \) and \( R_{k-1} = |R_k - r_k| \) are the distances from the transmitter and the receiver to the reflection point, \( r_k \), in the plane. The parameter \( \Phi \) here denotes the double variant characteristic function of the large-scale surface elevation \( \zeta \) in Fig. 1:

\[
\Phi(r_k, r_{k-1}) = \exp(-i\kappa_{z,k} \zeta(r_k) + i\kappa_{z,k-1} \zeta(r_{k-1}))
\]

Following the analytical procedure in Refs. [26,33] and the conventional geometric optics model applied to the bistatic GPS scattering from the randomly rough sea-surface [34], the exponential terms in Eq. (A1) can be simplified using a Taylor expansion over \( r_k - r_{k-1} \). The result can then be represented as:

\[
Y_k(t, f_D) \cdot Y_{k-1}(t, f_D) = T_i^2 \frac{16\pi}{2} \int \int \frac{d_k d_{k-1} A^2(\Delta r)}{R_k^2(\rho) R_{k-1}^2(\rho)} \times \\
|\sin c(2\pi f T_i)|^2 D^2(\rho) |\sigma(\rho)| d^2 \rho \quad \text{(A3)}
\]

where \( \rho = (r_k + r_{k-1})/2 \) is a new sea surface position vector and

\[
\sigma(\rho) = \frac{|\mathbf{R}|^2 q^4(\rho)}{4\pi q_z^2(\rho)} \int \exp[-i\kappa_{\perp}(\rho) \cdot (r_k - r_{k-1})] \times \\
\Phi[(r_k - r_{k-1}) \cdot \rho]^2 (r_k - r_{k-1}) = \\
\frac{\pi |\mathbf{R}|^2 q^4(\rho)}{q_z^2(\rho)} P_{\nu} \left( -\frac{\kappa_{\perp}}{q_z} \right) \quad \text{(A4)}
\]
stands for the normalized bistatic scattering cross section where $P_n$ is the probability density function of the sea surface slope.

The simulated DDMs are efficiently computed from the result of Eq. (A3) using fast-Fourier transforms rewritten in the form of the so-called radar mapping equation as:

$$
(Y_k(\tau, f_D) \cdot Y^*_k(\tau, f_D)) = \Sigma(\tau, f_D) \ast * \delta_k d_k^{-1} \Lambda^2(\Delta \tau) \sin c(2\pi \Delta f T_i))^2
\tag{A5}
$$

where $\Sigma$ represents complex factors for the sea-surface geometry with the terms from the bistatic radar equation and antenna radiation pattern being

$$
\Sigma(\tau, f_D) = \frac{T^2}{4\pi} \int \frac{D^2(\rho)\sigma_0(\rho)}{R^2_1(\rho)R^2_2(\rho)} \delta(\tau - \rho) \delta(f_d - f_D(\rho)) d^2\rho
\tag{A6}
$$

where $\delta$ is the Dirac delta function. In practical receivers, the DDMs are not calculated over the $x-y$ coordinates as in Fig. 1, but are computed over the $(\tau, f_D)$ domain after the transformation \cite{35}:

$$
\begin{align*}
\tau &= \tau(x, y), \\
\rho &= f_d(x, y)
\end{align*}
\tag{A7}
$$

Then $\Sigma(\tau, f_D)$ becomes

$$
\Sigma(\tau, f_D) = \frac{T^2}{4\pi} \int \frac{D^2(\rho, f_D)\sigma_0(\rho)}{R^2_1(\rho, f_D)R^2_2(\rho, f_D)} \delta(\tau - \rho) \delta(f_d - f_D(\rho)) J(\tau, f_D) d\rho
\tag{A8}
$$

where $J(\tau, f_D)$ denotes the Jacobian of the variables defined in Eq. (A7). Using the properties of the $\delta$ function and substituting Eq. (A8) into Eq. (A5) yields the final expression for the power distribution:

$$
(Y_k(\tau, f_D) \cdot Y^*_k(\tau, f_D)) = \delta_k d_k^{-1} \Lambda^2(\Delta \tau) \sin c(2\pi \Delta f T_i))^2 \ast *
\frac{T^2}{4\pi} \frac{D^2(\rho, f_D)\sigma_0(\rho, f_D)}{R^2_1(\rho, f_D)R^2_2(\rho, f_D)} J(\tau, f_D)
\tag{A9}
$$

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References


Yiran Wang was born in Liaoning, China. He received the BEng degree from the Beihang University (formerly Beijing University of Aeronautics and Astronautics), Beijing, China in 2008. He is currently working toward the PhD degree in Beihang University. He is with School of Electronic Information Engineering, Beihang University. His current research activities are related to signal processing of GNSS-R.

Bo Zhang was born in Shandong, China. He received the PhD degree from the Beihang University (formerly Beijing University of Aeronautics and Astronautics), Beijing, China in 2006. He is working now as a lecturer in Beihang University. His current research activities are related to signal processing of GNSS.

Dingrong Shao received the BS degree in radio communications from Beihang University (formerly Beijing University of Aeronautics and Astronautics), Beijing, China, in 1962. Since 1981, he has been with the School of Electronics and Information Engineering, Beihang University, as a full professor. His research interests include spread-spectrum communications, signal processing for acousto-optic devices, and satellite navigation.