Tsinghua Science and Technology

Volume 18 | Issue 1 Article 7

2013

Low Complexity LDPC Decoder with Modified Sum-Product Algorithm

Chen Qian
Tsinghua National Laboratory for Information Science and Technology, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

Weilong Lei
Tsinghua National Laboratory for Information Science and Technology, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

Zhaocheng Wang
Tsinghua National Laboratory for Information Science and Technology, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

Recommended Citation
Chen Qian, Weilong Lei, Zhaocheng Wang. Low Complexity LDPC Decoder with Modified Sum-Product Algorithm. Tsinghua Science and Technology 2013, 18(1): 57-61.

TSINGHUA SCIENCE AND TECHNOLOGY
ISSN 1007-0214 07/12 pp57-61
Volume 18, Number 1, February 2013

Low Complexity LDPC Decoder with Modified Sum-Product Algorithm

Chen Qian, Weilong Lei, and Zhaocheng Wang*

Abstract: This paper describes an efficient implementation of the Sum-Product Algorithm (SPA) within a Low Density Parity Check (LDPC) code decoder, where a horizontal process correction term is used to improve the decoding performance of the Min-Sum algorithms. The correction term is implemented as a look-up table. The algorithm uses the correction term redundancy by means of a coordinate transformation to reduce the hardware complexity. Simulations and hardware tests indicate that the decoding performance is very good with the appropriate look-up table.

Key words: Low Density Parity Check (LDPC) code; decoding; look-up table

1 Introduction

The Low Density Parity Check (LDPC) code was first proposed by Gallager in 1962[1], but it did not get much attention until it was rediscovered by MacKay and Neal in 1997[2]. With its excellent error correcting performance, the LDPC code has attracted much attention from academia and industry and has been adopted by many commercial systems such as the Chinese Digital Terrestrial Multimedia Broadcasting (DTMB) standard and the European second generation Digital Video Broadcasting (DVB-T2) standard[3-5].

The LDPC code belongs to a class of linear block codes which use a sparse matrix as its parity check matrix. Decoder implementation uses a variety of decoding algorithms and different algorithms with different decoding results[6]. The most commonly used algorithm is the Sum-Product Algorithm (SPA)[7] and its simplified versions.

SPA is an optimal decoding algorithm for high coding rates. This iterative algorithm uses information exchange and updates between information points and check points to improve decoding performance and reduce the bit-error rate. Although the SPA has superior performance, its complexity, particularly for horizontal processes, limits applications in practice. The complexity of the SPA algorithm can be reduced by the Min-Sum algorithm[8,9]. The Min-Sum algorithm simplifies the horizontal process of the SPA, which significantly reduces the hardware complexity with some performance degradation. Several modified Min-Sum methods are proposed to reduce the performance losses[10], including the normalized Min-Sum and the offset Min-Sum algorithm. Those modified methods require more hardware resources compared with the Min-Sum algorithm, but their performance is closer to that of SPA.

This paper makes additional approximations to the horizontal process of the SPA. A correction term is introduced to improve the performance of the decoding process for limited hardware resource.

• Chen Qian, Weilong Lei, and Zhaocheng Wang are with Tsinghua National Laboratory for Information Science and Technology, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China. E-mail: qc8802@gmail.com; leiwl@mail.tsinghua.edu.cn; zcwang@mail.tsinghua.edu.cn.
∗ To whom correspondence should be addressed.
Manuscript received: 2012-02-17; accepted: 2012-12-10

2 SPA and Simplified SPA

The SPA enhances the reliability of the decoding process through the transmission and update of information between information points and check points. In general, the intermediate processes and computations are simplified by soft information expressed as a Log-Likelihood Ratio (LLR).

The LLR of bit b can be expressed as

$$\text{LLR}(b) = \ln\left(\frac{P(b=0)}{P(b=1)}\right) \tag{1}$$

Every iterative SPA process includes a horizontal process and a vertical process. The horizontal process updates information transferred from information points to check points. The soft information, L_{mn}, transferred from information point n to check point m is calculated as

$$L_{mn} = 2\tanh^{-1}\left(\prod_{n'\in N/n} \tanh(Z_{mn'}/2)\right) \tag{2}$$

where $Z_{mn'}$ denotes the soft information transferred from check points m to information points n', N indicates the node set of all information points, and N/n is a node set including all nodes in node set N excluding node n.

The vertical process updates information transferred from check points to information points. The soft information, Z_{mn}, transferred from check point m to information point n is given by

$$Z_{mn} = C_n + \sum_{m'\in M/m} L_{m'n} \tag{3}$$

where C_n is the channel information for information points n, M is the node set of all the check points, and M/m is a node set including all nodes in node set M excluding node m.

The condition for ending the iterations is that the hard decision of the soft information Z_{mn} satisfies the parity-check function of the LDPC codes.

SPA has outstanding performance but with high complexity. The horizontal process calculates the hyperbolic tangent function and the hyperbolic arc tangent function. The high complexity makes it difficult to be implemented by real hardware.

The Min-Sum algorithm simplifies the horizontal process as follows,

$$L_{mn} = \text{sign}\left(\prod_{n'\in N/n} Z_{mn'}\right) \min_{n'\in N/n}(|Z_{mn'}|) \tag{4}$$

This simplification uses only modulo and minimum operations. Thus the complexity of the Min-Sum algorithm is significantly less than that of the SPA and the Min-Sum algorithm is more suitable for hardware implementation. However, this simplified structure brings some loss of performance. The performance loss can be alleviated by some modifications, e.g., multiplied by a normalization factor[11] or subtracting an offset[12]. Appropriate selection of the normalization factor or the offset makes the decoding performance close to that of the SPA.

3 SPA Look-up Table

A correction term is introduced to the Min-Sum algorithm to improve the decoding performance by reducing the approximation error in the horizontal process. This section describes the characteristics of the correction term based on the differences between Eq. (2) and Eq. (4).

The horizontal process can be simplified to a process with only two inputs. With n inputs in the horizontal process,

$$y = 2\tanh^{-1}(\tanh(x_1/2)\tanh(x_2/2)\cdots\tanh(x_n/2)) \tag{5}$$

Define

$$y_1 = 2\tanh^{-1}(\tanh(x_1/2)\tanh(x_2/2)\cdots\tanh(x_{n-1}/2)) \tag{6}$$

as the horizontal process result for the first $n-1$ inputs. Substituting y_1 into Eq. (5) gives

$$y = 2\tanh^{-1}(\tanh(y_1/2)\tanh(x_n/2)) \tag{7}$$

Thus, the results with multiple inputs can be calculated recursively from results of two inputs.

Since both the hyperbolic tangent and the hyperbolic arc tangent are odd functions, we need only study the situation where the two inputs are not less than zero, that is

$$y = 2\tanh^{-1}(\tanh(x_1/2)\tanh(x_2/2)) \tag{8}$$

where $x_1 \geqslant 0$ and $x_2 \geqslant 0$.

For two inputs, Eq. (8) can be expressed as

$$y = \min(x_1, x_2) + f_c(x_1, x_2) \tag{9}$$

The correction term $f_c(x_1, x_2)$ can be given by a two-dimensional table in hardware. In addition, the calculations can be simplified by assuming that the dynamic range of the two inputs x_1 and x_2 are both $[0, 16]$. During hardware implementation, x_1 and x_2 are quantized by 8 bits, where the first bit is the sign bit.

3.1 Features of the correction term

Figure 1 shows contours of the difference between Eqs. (2) and (4) which is the correction term. The correction term is normally quite small, which means that it can be quantized using fewer bits without significant loss of accuracy. Furthermore, the correction term is always less than zero, which means the algorithm need only consider its absolute value without its sign information. However, the input ranges can not be reduced; thus more bits are required to quantize the inputs with this correction term. Finally, there are many zeros and many points having the same value. This redundancy in the correction term can be used to reduce the complexity of the look-up table.

3.2 Calculation of the correction term

The redundancy in the correction term is used to simplify the implementation of the look-up table by using a coordinate transformation. Figure 1 shows that correction term is symmetric along the line of $x_1 = x_2$, which implies that a coordinate transformation can reduce the expression of the correction term in hardware. One coordinate transformation is

$$\begin{cases} x'_1 = \min(x_1, x_2), \\ x'_2 = |x_2 - x_1| \end{cases} \tag{10}$$

The correction term after the coordinate transformation is shown in Fig. 2, which is more suitable for hardware implementation.

The values of the correction term are almost the same for a given x'_1 for large x'_2 and the correction term is nearly zero for large x'_1. Therefore, after the

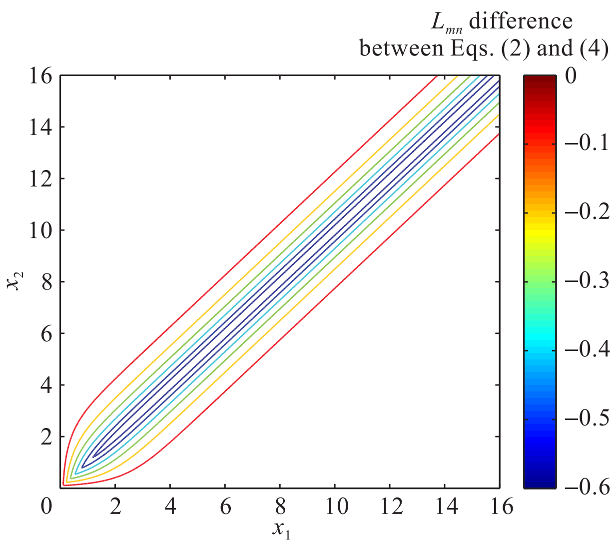

Fig. 1 Correction term.

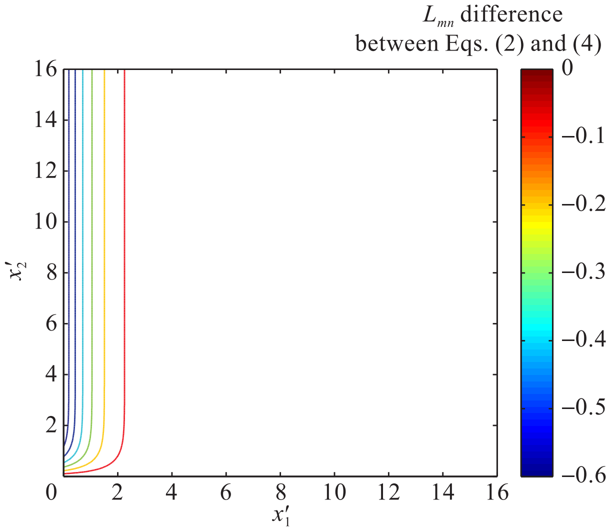

Fig. 2 Correction term after the coordinate transformation.

coordinate transformation, the quantization error will be smaller when the same number of bits are used to quantize the correction term. Thus, fewer bits can be used to quantize the correction term without affecting the quantization error. The horizontal process result is then found by adding the correction term to the smaller of the two inputs, which can be easily obtained during the coordinate transformation.

3.3 Two-input Horizontal Process Unit (HPU) structure

The two-input HPU for the SPA look-up table method includes several parts:

(1) The coordinate transformation unit calculates the two indexes for the two-dimensional look-up table;

(2) A search of the look-up table then gives the correction term using the two inputs after the coordinate transformation as indexes;

(3) The post-processing unit calculates the final output of the two-input HPU by adding the correction term to the smaller of the two inputs obtained from the coordinate transformation.

The output of a multiple-input HPU is then obtained recursively using the outputs of a two-input HPU.

4 Simulation Results

Simulations using the SPA look-up table method were used to compare the results of the SPA and the normalized Min-Sum algorithm.

The simulation conditions are as follows.

(1) A quasi-cyclic LDPC code with rate 2/3 and length 61 440. The LDPC code design[13,14] used column weights of 13, 3, and 2 with the proportions

of 1/15, 9/15, and 5/15. The row weights were 9, 10, and 11, so 9-input, 10-input, and 11-input HPU were needed. SPA was used as the decoding algorithm with a maximum of 30 iterations;

(2) The constellation mapping was 64QAM with Gray mapping. The demapping algorithm used the Log-MAP algorithm, the best demapping algorithm when additive white Gaussian noise channel is present;

(3) The channel model was to be a Rayleigh channel;

(4) Bit-interleaving was used between the encoding and constellation mapping with corresponding bit-deinterleaving used between the decoding and demapping. The interleaver pattern was defined in Chinese Digital Terrestrial Television Broadcasting Standard[3];

(5) The simulations used 8.192×10^8 bits.

Figure 3 shows that, for a bit error rate of about 10^{-5}, 3 bits could be used for quantizing the two HPU inputs (so the two-dimensional look-up table was 8×8) with a dynamic input range of $[0, 16]$. The look-up table method then gave only a 0.025 dB gain compared with the normalized Min-Sum algorithm. However, if 4 bits were used for the quantization with a 16×16 look-up table, the look-up table method performance was close to that of the SPA with only a 0.01 dB difference between the two algorithms. This gives about 0.1 dB gain compared with the normalized Min-Sum algorithm.

These simulation results were confirmed by implementing LDPC decoders using the look-up table and the Min-Sum method are implemented. The test conditions were in the following:

(1) The LDPC code was based on the Chinese Digital Terrestrial Television Broadcasting Standard[3] with a rate of 0.6. The normalized Min-Sum algorithm was used as the decoding algorithm with a maximum of 30 iterations;

(2) The constellation mapping used 64QAM with the MAX-Log-MAP algorithm used for demapping;

(3) The channel model was the AWGN channel;

(4) The decoding algorithm used the normalized Min-Sum algorithm and the SPA look-up table method with an 8×8 two-dimensional look-up table;

(5) The tests used about 300 million bits.

Figure 4 shows that for an AWGN channel with an 8×8 look-up tables and a bit error rate of about 10^{-5}, the look-up table gave about 0.05 dB over the normalized Min-Sum LDPC decoding algorithm.

5 Conclusions

This paper briefly describes a modified Min-Sum algorithm using an SPA look-up table to simplify the Sum-Product algorithm. A coordinate transform is used to simplify the correction term for the horizontal process to improve the decoding with less complexity. Simulations and hardware tests show that this algorithm improves the decoding performance with the appropriate look-up table.

Acknowledgements

This work was supported by Tsinghua University Initiative Scientific Research Program (No. 2010THZ0) and the National Key Basic Research (973) Program of China (No. 2012CB316000).

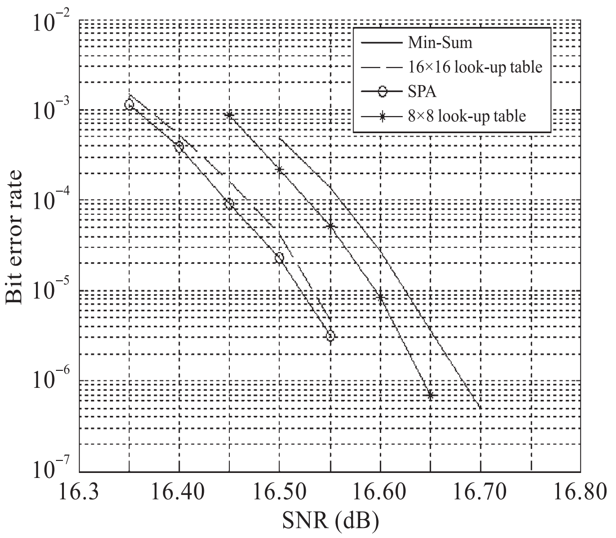

Fig. 3 Simulation results.

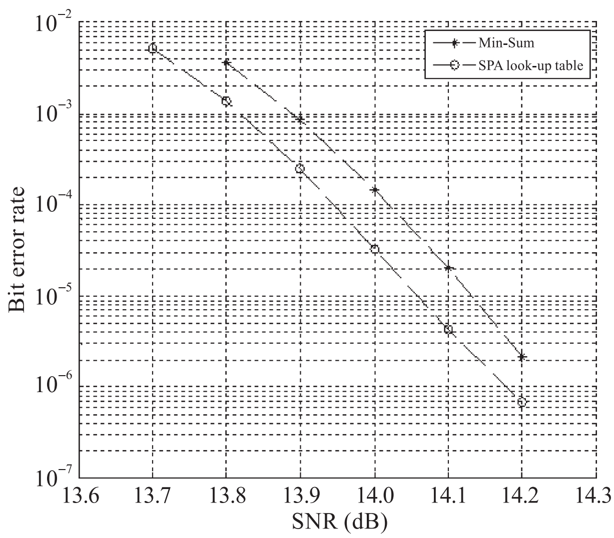

Fig. 4 Hardware test results.

References

[1] R. G. Gallager, Low-density parity-check codes, *IEEE Trans. Inf. Theory*, vol. 8, no. 1, pp. 21-28, Jan. 1962.

[2] D. J. MacKay and R. M. Neal, Near shannon limit performance of low-density parity check codes, *Electron. Lett.*, vol. 33, no. 6, pp. 457-458, Mar. 1997.

[3] *Framing Structure, Channel Coding and Modulation for Digital Television Terrestrial Broadcasting System*, (in Chinese), GB 20600-2006, Standardization Administration of the People's Republic of China, 2006.

[4] Digital Video Broadcasting (DVB), Frame structure channel coding and modulation for a second generation digital terrestrial television broadcasting system (DVB-T2), ETSI EN 302 755 v1.1.1, Sept. 2009.

[5] L. Dai, Z. Wang, and Z. Yang, Next-generation digital television terrestrial broadcasting systems: Key technologies and research trends, *IEEE Commun. Magazine*, vol. 50, no.6, pp. 150-158, Jun. 2012.

[6] W. E. Ryan and S. Lin, *Channel Codes: Classical and Modern*, New York, NY, USA: Cambridge University Press, 2009.

[7] D. J. MacKay, Good error-correcting codes based on very sparse matrices, *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 399-431, Mar. 1999.

[8] M. P. C. Fossorier, M. Mihaljevic, and H. Imai, Reduced complexity iterative decoding of low-density parity check codes based on belief propagation, *IEEE Trans. Commun.*, vol. 47, no. 5, pp. 673-680, May 1999.

[9] A. J. Felstrom and K. S. Zigangirov, Time-varying periodic convolutional codes with low-density parity-check matrix, *IEEE Trans. Inf. Theory*, vol. 45, no.6, pp. 2181-2191, Sep. 1999.

[10] J. Chen, Reduced complexity decoding algorithms for low-density parity check codes and turbo codes, Ph.D. dissertation, Dept. Electrical Engineering, The University of Hawaii, Honolulu, USA, 2003.

[11] J. Chen and M. P. C. Fossorier, Near optimum universal belief propagation based decoding of low-density parity check codes, *IEEE Trans. Commun.*, vol. 50, no. 3, pp. 406-414, Mar. 2002.

[12] J. Chen, A. Dholakia, E. Eleftheriou, M. P. C. Fossorier, and X. Hu, Reduced-complexity decoding of LDPC codes, *IEEE Trans. Commun.*, vol. 53, no. 7, pp. 1232, Jul. 2005.

[13] X. Hu and D. M. Arnold, Regular and irregular progressive edge-growth tanner graphs, *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 386-389, Jan. 2005.

[14] I. Djurdjevic, J. Xu, K. A. Ghaffar, and S. Lin, A class of low-density parity-check codes constructed based on reed-solomon codes with two information symbols, *IEEE Commun. Lett.*, vol. 7, no. 7, pp. 317-319, Jul. 2003.

Chen Qian received his BEng degree in electronic engineering from Tsinghua University in 2010 and He is now a PhD candidate of Tsinghua University. His research area includes channel coding technology and MIMO detection technology.

Weilong Lei received the BEng degree in telecommunication engineering from Beijing Jiaotong University, Beijing, China, in 2005 and MEng degree in electronic engineering from Tsinghua University, Beijing, China, in 2008. He worked as a research engineer from 2008 at DTV center in Tsinghua University. His research areas include wireless communications and digital broadcasting and his interests focus on channel estimation and channel coding.

Zhaocheng Wang received his BEng, MEng, and PhD degrees from Tsinghua University in 1991, 1993, and 1996, respectively. From 1996 to 1997, he was with Nanyang Technological University (NTU) in Singapore as a Post Doctoral Fellow. From 1997 to 1999, he was with OKI Techno Centre (Singapore) Pte. Ltd., firstly as a research engineer and then as a senior engineer. From 1999 to 2009, he worked at SONY Deutschland GmbH, firstly as a senior engineer and then as a principal engineer. He is currently a Professor at the Department of Electronic Engineering, Tsinghua University. His research areas include wireless communications, digital broadcasting, and millimeter wave communications. He holds 29 granted US/EU patents and has published over 70 technical papers. He has served as technical program committee co-chair/member of many international conferences. He is a senior member of IEEE and a fellow of IET.