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SHORT COMMUNICATION

Transformation between polar and rectangular coordinates of stiffness and dampness parameters in hydrodynamic journal bearings

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Abstract: The stiffness and dampness parameters of journal bearings are required in rectangular coordinates for analyzing the stability boundary and threshold speed of oil film bearings. On solving the Reynolds equation, the oil film force is always obtained in polar coordinates; thus, the stiffness and dampness parameters can be easily obtained in polar coordinates. Therefore, the transformation between the polar and rectangular coordinates of journal bearing stiffness and dampness parameters is discussed in this study.

Keywords: coordinate transformation; stiffness parameters; dampness parameters; hydrodynamic journal bearings

1 Introduction

In modern industry, rotating parts of engineering equipment are supported by journal bearings in the vertical direction. The machine characteristics are significantly dependent on the performances of journal bearings. Due to the oil whip effect in a rotating hydrodynamic journal bearing, self-excited vibration occurs in the oil film, which increases with an increase in the rotation speed [1–4]. As a result of the self-excited vibration, threshold speed and stability boundary exist for the rotating bearings. When the rotating speed is larger than the threshold speed, the vibration results in large orbiting amplitudes of the journal and leads to the contact between the journal and bearing, causing bearing failure. Further, the bearing is stable at the journal bearing center in the stability boundary. Khonsari and Chang [5] analyzed the nonlinear stability of journal bearings, and obtained the stability boundary by tracking the journal center trajectory. To easily derive the linear threshold speed, Huang et al. [6] chose polar coordinates instead of Cartesian coordinates to signify the state vector. Lin et al. [7, 8] discussed the threshold speed and stability boundary of hydrodynamic journal bearings lubricated using non-Newtonian fluids. Kushare and Sharma [9] dealt with the threshold speed of two lobe symmetric hole entry worn hybrid journal bearing by theoretically considering the non-Newtonian behavior of the lubricants.

The threshold speed and stability boundary of the hydrodynamic journal bearing are determined based on the stiffness and damping coefficients. Lund and Thomsen [10] proposed a method to study these coefficients. The Reynolds equation is derived with respect to Cartesian coordinates and stiffness and damping coefficients are obtained by integrating the new equations. Based on this method, Jang and Kim [11] studied the dynamic characteristics of journal bearings with five degrees of freedom (DOF) and derived the perturbation equations. Crooijmans et al. [12] discussed the self-excited vibration of hydrodynamic journal bearings, and interpreted the dynamic performance

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List of symbols

C	radial clearance, μm
D	journal diameter, mm
e	journal eccentricity, mm
f_ε	fluid film reaction component on the eccentric direction, N
f_φ	fluid film reaction component perpendicular to the eccentric direction, N
F_X	fluid film reaction component on X direction, N
F_Y	fluid film reaction component on Y direction, N
h	film thickness, mm
L	bearing length, mm
m	mass of rotor per each bearing, kg
p	pressure, $N \cdot m^{-2}$
R	radius of journal, mm
W	external load, N
x	coordinate of circumferential direction, mm
y	coordinate of the eccentric direction, mm
z	coordinate of axial direction, mm
X, Y, Z	Cartesian coordinates

Greek symbols

ε	eccentricity ratio, e/C
θ	angular coordinate, °
μ	viscosity of zero shear rate, $N \cdot s \cdot m^{-2}$
φ	attitude angle, rad
ω	rotation speed of shaft, rad/s
$\omega^* =$	$\omega(W/mC)^{1/2}$
ω_s^*	non-dimensional stability threshold speed

Non-dimensional parameters

C_{ij}^*	non-dimensional damping coefficients, $i, j = X, Y$
$f_\varepsilon^* =$	$f_\varepsilon/(SW)$
$f_\varphi^* =$	$f_\varphi/(SW)$
$F_X^* =$	F_X/W
$F_Y^* =$	F_Y/W
$h^* =$	h/c
K_{ij}^*	non-dimensional stiffness coefficients, $i, j = X, Y$
$p^* =$	$pC^2/(\mu\omega R^2)$
$S =$	$\mu\omega RL^3/(WC^2)$
$z^* =$	$2z/L$

Subscripts and superscripts

gz	gradient on z direction
s	stability state
ε	component on eccentric direction
φ	component perpendicular to eccentric direction
*	non-dimensional parameter

of rotor-bearing systems. Wang and Khonsari [13] proposed a method to calculate the dynamic coefficients to obtain an improved physical meaning. It is known that oil stiffness and damping characteristics are important for linear threshold speed and stability boundary of hydrodynamic bearings.

The coordinate system applied in the general Reynolds equation is polar coordinates. Thus, the oil film force is always obtained in polar coordinates and the stiffness and dampness parameters can be easily obtained in polar coordinates. While discussing linear threshold speed and stability boundary of hydrodynamic bearings, the stiffness and dampness parameters in rectangular coordinates are required. As per our understanding, the coordinate system applied in the previous study on linear threshold speed and stability boundary was either polar coordinates or rectangular coordinates for the complete calculation, which complicated the analysis. To address this complication, the transformation between polar and rectangular coordinates of journal bearing stiffness and dampness parameters is discussed in this study. The stiffness and dampness parameters are first calculated in polar coordinates, and then, converted into rectangular coordinates. Further, the stiffness and dampness parameters in rectangular coordinates can be applied to calculate linear threshold speed and stability boundary of hydrodynamic bearings.

2 Transformation between polar and rectangular coordinates

Figure 1 shows the journal bearing structure. The journal rotates in an anticlockwise direction with an angular speed ω and a radius R. The radial clearance is expressed as C, eccentricity is expressed as e, angular

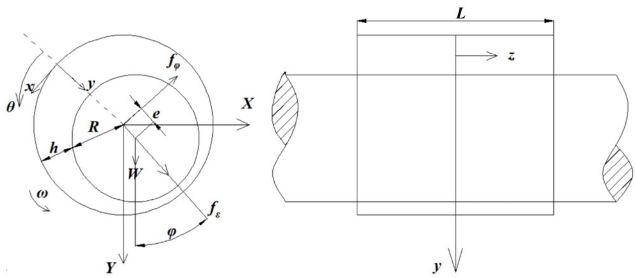

Fig. 1 Schematic diagram of the journal bearing.

coordinate of the bearing surface is expressed as θ, and film thickness is expressed as $h = C + e\cos\theta$. Further, φ is the attitude angle, f_φ is the fluid force component in the direction of attitude angle, and f_ε is a component in the eccentric direction. The external load on the journal is W.

The non-dimensional parameters are as follows:

$$x = R\theta, z = \frac{L}{2}z^*, \alpha = k\left(\frac{\mu R\omega}{C}\right)^2, h = Ch^* = C\left(1+\varepsilon\cos\theta\right),$$
$$e = C\varepsilon, p = \mu\omega\left(\frac{R}{C}\right)^2 p^*, \beta = \frac{D}{L},$$
$$\dot{\varepsilon} = \frac{1}{\omega}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}, \dot{\varphi} = \frac{1}{\omega}\frac{\mathrm{d}\varphi}{\mathrm{d}t}, S = \frac{\mu\omega R^3 L}{WC^2}, f_\varepsilon = \frac{\mu\omega R^3 L}{C^2}f_\varepsilon^*,$$
$$f_\varphi = \frac{\mu\omega R^3 L}{C^2}f_\varphi^*, F_X = F_X^* W, F_Y = F_Y^* W \tag{1}$$

Threshold speed is an important parameter for the stability of the oil film bearing, and the non-dimensional linear threshold speed of the journal bearing ω_s^* is as follows [5, 8].

$$\omega_s^* = \sqrt{\frac{\dfrac{C_{XX}^*C_{YY}^* - C_{XY}^*C_{YX}^*}{\dfrac{K_{XX}^*C_{YY}^* + K_{YY}^*C_{XX}^* - K_{XY}^*C_{YX}^* - K_{YX}^*C_{XY}^*}{C_{XX}^* + C_{YY}^*} - \left(K_{XX}^* + K_{YY}^*\right) + \dfrac{K_{XX}^*K_{YY}^* - K_{XY}^*K_{YX}^*}{K_{XX}^*C_{YY}^* + K_{YY}^*C_{XX}^* - K_{XY}^*C_{YX}^* - K_{YX}^*C_{XY}^*}}} \tag{2}$$

where K_{ij}^* and C_{ij}^* ($i, j = X, Y$) are the non-dimensional stiffness and dampness parameters of the bearing in rectangular coordinates, respectively.

Although the results of oil film forces in journal bearings have always been expressed in polar coordinates, in this study, the transformation between polar and rectangular coordinates of journal bearing stiffness and dampness parameters is discussed as follows:

The non-dimensional oil film forces are expended in the X and Y directions as a first-order Taylor series approximation, as shown in Eq. (3).

$$\begin{bmatrix}\mathrm{d}F_X^*\\ \mathrm{d}F_Y^*\end{bmatrix} = -\boldsymbol{K}_\mathrm{r}\begin{bmatrix}\mathrm{d}X^*\\ \mathrm{d}Y^*\end{bmatrix} - \boldsymbol{C}_\mathrm{r}\begin{bmatrix}\mathrm{d}\dot{X}^*\\ \mathrm{d}\dot{Y}^*\end{bmatrix} \tag{3}$$

where $\boldsymbol{K}_\mathrm{r}$ and $\boldsymbol{C}_\mathrm{r}$ are the non-dimensional stiffness and dampness matrixes in rectangular coordinates, respectively. The expressions for $\boldsymbol{K}_\mathrm{r}$ and $\boldsymbol{C}_\mathrm{r}$ are as follows:

$$\boldsymbol{K}_\mathrm{r} = \begin{bmatrix}K_{XX}^* & K_{XY}^*\\ K_{YX}^* & K_{YY}^*\end{bmatrix} = \begin{bmatrix}\dfrac{\partial F_X^*}{\partial X^*} & \dfrac{\partial F_X^*}{\partial Y^*}\\ \dfrac{\partial F_Y^*}{\partial X^*} & \dfrac{\partial F_Y^*}{\partial Y^*}\end{bmatrix},$$
$$\boldsymbol{C}_\mathrm{r} = \begin{bmatrix}C_{XX}^* & C_{XY}^*\\ C_{YX}^* & C_{YY}^*\end{bmatrix} = \begin{bmatrix}\dfrac{\partial F_X^*}{\partial \dot{X}^*} & \dfrac{\partial F_X^*}{\partial \dot{Y}^*}\\ \dfrac{\partial F_Y^*}{\partial \dot{X}^*} & \dfrac{\partial F_Y^*}{\partial \dot{Y}^*}\end{bmatrix} \tag{4}$$

In Fig. 1, the non-dimensional resultant forces in the X and Y directions are as follows:

$$F_X^*\left(X^*, Y^*, \dot{X}^*, \dot{Y}^*\right) = S\left(f_\varepsilon^*\sin\varphi + f_\varphi^*\cos\varphi\right) \tag{5}$$

$$F_Y^*\left(X^*, Y^*, \dot{X}^*, \dot{Y}^*\right) = S\left(f_\varepsilon^*\cos\varphi - f_\varphi^*\sin\varphi\right) + 1 \tag{6}$$

where S is the Sommerfeld number of the bearing. At the equilibration position, the expression of S is

$$S = \left(\frac{1}{\sqrt{f_\varepsilon^{*2} + f_\varphi^{*2}}}\right)_s \tag{7}$$

where the subscript s denotes the equilibrium position.

The differential of Eqs. (5) and (6) is calculated as

$$\begin{bmatrix}\mathrm{d}F_X^*\\ \mathrm{d}F_Y^*\end{bmatrix} = S\boldsymbol{A}\left(\begin{bmatrix}\mathrm{d}f_\varepsilon^*\\ \mathrm{d}f_\varphi^*\end{bmatrix} - \begin{bmatrix}f_\varphi^*\mathrm{d}\varphi\\ -f_\varepsilon^*\mathrm{d}\varphi\end{bmatrix}\right) \tag{8}$$

where $\boldsymbol{A}$ is a transformation matrix as follows:

$$\boldsymbol{A} = \begin{bmatrix}\sin\varphi & \cos\varphi\\ \cos\varphi & -\sin\varphi\end{bmatrix} \tag{9}$$

The differential of non-dimensional oil film forces in the ε and φ directions are

$$\begin{bmatrix} \mathrm{d}f_\varepsilon^* \\ \mathrm{d}f_\varphi^* \end{bmatrix} = \boldsymbol{K}_p \begin{bmatrix} \mathrm{d}\varepsilon \\ \varepsilon \mathrm{d}\varphi \end{bmatrix} + \boldsymbol{C}_p \begin{bmatrix} \mathrm{d}\dot{\varepsilon} \\ \varepsilon \mathrm{d}\dot{\varphi} \end{bmatrix} \tag{10}$$

where $\boldsymbol{K}_p$ and $\boldsymbol{C}_p$ are the non-dimensional stiffness and dampness matrixes in rectangular coordinates, respectively. The expressions of $\boldsymbol{K}_p$ and $\mathbf{C}_p$ are

$$\boldsymbol{K}_p = \begin{bmatrix} \dfrac{\partial f_\varepsilon^*}{\partial \varepsilon} & \dfrac{\partial f_\varepsilon^*}{\varepsilon \partial \varphi} \\ \dfrac{\partial f_\varphi^*}{\partial \varepsilon} & \dfrac{\partial f_\varphi^*}{\varepsilon \partial \varphi} \end{bmatrix}, \quad \boldsymbol{C}_p = \begin{bmatrix} \dfrac{\partial f_\varepsilon^*}{\partial \dot{\varepsilon}} & \dfrac{\partial f_\varepsilon^*}{\varepsilon \partial \dot{\varphi}} \\ \dfrac{\partial f_\varphi^*}{\partial \dot{\varepsilon}} & \dfrac{\partial f_\varphi^*}{\varepsilon \partial \dot{\varphi}} \end{bmatrix} \tag{11}$$

By substituting Eq. (10) in Eq. (8), we get

$$\begin{bmatrix} \mathrm{d}F_X^* \\ \mathrm{d}F_Y^* \end{bmatrix} = S \begin{bmatrix} \sin\varphi & \cos\varphi \\ \cos\varphi & -\sin\varphi \end{bmatrix} \left\{ \left(\boldsymbol{K}_p + \boldsymbol{K}_{p0} \right) \begin{bmatrix} d\varepsilon \\ \varepsilon d\varphi \end{bmatrix} + \boldsymbol{C}_p \begin{bmatrix} \mathrm{d}\dot{\varepsilon} \\ \varepsilon \mathrm{d}\dot{\varphi} \end{bmatrix} \right\} \tag{12}$$

where $\boldsymbol{K}_{p0}$ is the non-dimensional stiffness correction matrix as follows:

$$\boldsymbol{K}_{p0} = \begin{bmatrix} 0 & -\dfrac{f_\varphi^*}{\varepsilon} \\ 0 & \dfrac{f_\varepsilon^*}{\varepsilon} \end{bmatrix} \tag{13}$$

The coordinate transformations between (X^*, Y^*) and (ε, φ) at the equilibrium position are

$$\begin{bmatrix} \mathrm{d}X^* \\ \mathrm{d}Y^* \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \mathrm{d}\varepsilon \\ \varepsilon \mathrm{d}\varphi \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \mathrm{d}\dot{X}^* \\ \mathrm{d}\dot{Y}^* \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \mathrm{d}\dot{\varepsilon} \\ \varepsilon \mathrm{d}\dot{\varphi} \end{bmatrix} \tag{15}$$

When Eqs. (14) and (15) are substituted in Eq. (12), the expression obtained is

$$\begin{bmatrix} \mathrm{d}F_X^* \\ \mathrm{d}F_Y^* \end{bmatrix}_s = S\boldsymbol{A} \left\{ \left(\boldsymbol{K}_p + \boldsymbol{K}_{p0} \right) \boldsymbol{A} \begin{bmatrix} \mathrm{d}X^* \\ \mathrm{d}Y^* \end{bmatrix} + \boldsymbol{C}_p \boldsymbol{A} \begin{bmatrix} \mathrm{d}\dot{X}^* \\ \mathrm{d}\dot{Y}^* \end{bmatrix} \right\} \tag{16}$$

From Eqs. (3) and (16), the stiffness and dampness coefficients of the oil film on Cartesian coordinate system at the equilibrium position are derived as follows:

$$\boldsymbol{K}_\mathrm{r} = -S\boldsymbol{A}\left(\boldsymbol{K}_p + \boldsymbol{K}_{p0} \right)\boldsymbol{A} \tag{17}$$

$$\mathbf{C}_r = -S\boldsymbol{A}\boldsymbol{C}_p\boldsymbol{A} \tag{18}$$

Equations (17) and (18) are the general transformation of the hydrodynamic journal bearing stiffness and dampness coefficients between polar and rectangular coordinates, respectively. The transformation validity for short journal bearings is discussed in Section 3.

3 Verification and discussion

When the short-bearing approximate is applied with half-Sommerfeld boundary conditions, the non-dimensional oil film forces of the bearing are as follows [6, 13].

$$f_\varepsilon^* = -\left(\frac{\varepsilon^2 \left(1 - 2\dot{\varphi}\right)}{\left(1 - \varepsilon^2\right)^2} + \frac{\pi\left(1 + 2\varepsilon^2\right)\dot{\varepsilon}}{2\left(1 - \varepsilon^2\right)^{\frac{5}{2}}} \right) \tag{19}$$

$$f_\varphi^* = \frac{\pi\varepsilon\left(1 - 2\dot{\varphi}\right)}{4\left(1 - \varepsilon^2\right)^{\frac{3}{2}}} + \frac{2\varepsilon\dot{\varepsilon}}{\left(1 - \varepsilon^2\right)^2} \tag{20}$$

At equilibrium position, the stiffness and dampness parameters of the bearing in polar coordinates are

$$K_{\varepsilon\varepsilon}^* = \frac{\partial f_\varepsilon^*}{\partial \varepsilon} = -\frac{2\varepsilon\left(1 + \varepsilon^2\right)}{\left(1 - \varepsilon^2\right)^3}, \quad K_{\varepsilon\varphi}^* = \frac{\partial f_\varepsilon^*}{\varepsilon \partial \varphi} - \frac{f_\varphi^*}{\varepsilon} = -\frac{\pi}{4\left(1 - \varepsilon^2\right)^{\frac{3}{2}}},$$

$$K_{\varphi\varepsilon}^* = \frac{\partial f_\varphi^*}{\partial \varepsilon} = \frac{\pi\left(1 + 2\varepsilon^2\right)}{4\left(1 - \varepsilon^2\right)^{\frac{5}{2}}}, \quad K_{\varphi\varphi}^* = \frac{\partial f_\varphi^*}{\varepsilon \partial \varphi} + \frac{f_\varepsilon^*}{\varepsilon} = -\frac{\varepsilon}{\left(1 - \varepsilon^2\right)^2} \tag{21}$$

$$D_{\varepsilon\varepsilon}^* = \frac{\partial f_\varepsilon^*}{\partial \dot{\varepsilon}} = -\frac{\pi\left(1 + 2\varepsilon^2\right)}{2\left(1 - \varepsilon^2\right)^{\frac{5}{2}}}, \quad D_{\varepsilon\varphi}^* = \frac{\partial f_\varepsilon^*}{\varepsilon \partial \dot{\varphi}} = \frac{2\varepsilon}{\left(1 - \varepsilon^2\right)^2},$$

$$D_{\varphi\varepsilon}^* = \frac{\partial f_\varphi^*}{\partial \dot{\varepsilon}} = \frac{2\varepsilon}{\left(1 - \varepsilon^2\right)^2}, \quad D_{\varphi\varphi}^* = \frac{\partial f_\varphi^*}{\varepsilon \partial \dot{\varphi}} = -\frac{\pi}{2\left(1 - \varepsilon^2\right)^{\frac{3}{2}}} \tag{22}$$

Further, the attitude angle at the equilibrium position is

$$\varphi = \tan^{-1}\left(\frac{f_\varphi^*}{f_\varepsilon^*} \right) = \tan^{-1}\left[\frac{\pi\left(1 - \varepsilon^2\right)}{4\varepsilon} \right] \tag{23}$$

When Eqs. (7) and (21)–(23) are substituted in Eqs. (17)–(18), the stiffness and dampness parameters in rectangular coordinates are

$$\begin{aligned}
K_{XX}^{*} &= -S\left(\sin^2\varphi K_{\varepsilon\varepsilon}^{*}+\sin\varphi\cos\varphi\left(K_{\varphi\varepsilon}^{*}+K_{\varepsilon\varphi}^{*}\right)+\cos^2\varphi K_{\varphi\varphi}^{*}\right)= \\
&\frac{4\left[2\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]}{\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}} \\
K_{XY}^{*} &= -S\left(\cos^2\varphi K_{\varphi\varepsilon}^{*}+\sin\varphi\cos\varphi\left(K_{\varepsilon\varepsilon}^{*}-K_{\varphi\varphi}^{*}\right)-\sin^2\varphi K_{\varepsilon\varphi}^{*}\right)= \\
&\frac{-\pi\left[-\pi^2+2\pi^2\varepsilon^2+\left(16-\pi^2\right)\varepsilon^4\right]}{\varepsilon\left(1-\varepsilon^2\right)^{\frac{1}{2}}\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}} \\
K_{YX}^{*} &= -S\left(\cos^2\varphi K_{\varepsilon\varphi}^{*}+\sin\varphi\cos\varphi\left(K_{\varepsilon\varepsilon}^{*}-K_{\varphi\varphi}^{*}\right)-\sin^2\varphi K_{\varphi\varepsilon}^{*}\right)= \\
&\frac{-\pi\left[\pi^2+\left(32+\pi^2\right)\varepsilon^2+2\left(16-\pi^2\right)\varepsilon^4\right]}{\varepsilon\left(1-\varepsilon^2\right)^{\frac{1}{2}}\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}} \\
K_{YY}^{*} &= -S\left(\cos^2\varphi K_{\varepsilon\varepsilon}^{*}-\sin\varphi\cos\varphi\left(K_{\varphi\varepsilon}^{*}+K_{\varepsilon\varphi}^{*}\right)+\sin^2\varphi K_{\varphi\varphi}^{*}\right)= \\
&\frac{4\left[\pi^2+\left(32+\pi^2\right)\varepsilon^2+2\left(16-\pi^2\right)\varepsilon^4\right]}{\left(1-\varepsilon^2\right)\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}}
\end{aligned} \tag{24}$$

$$\begin{aligned}
D_{XX}^{*} &= -S\left(\sin^2\varphi D_{\varepsilon\varepsilon}^{*}+\sin\varphi\cos\varphi\left(D_{\varphi\varepsilon}^{*}+D_{\varepsilon\varphi}^{*}\right)+\cos^2\varphi D_{\varphi\varphi}^{*}\right)= \\
&\frac{2\pi\left(1-\varepsilon^2\right)^{\frac{1}{2}}\left[\pi^2+2\left(\pi^2-8\right)\varepsilon^2\right]}{\varepsilon\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}} \\
D_{XY}^{*} &= -S\left(\cos^2\varphi D_{\varphi\varepsilon}^{*}+\sin\varphi\cos\varphi\left(D_{\varepsilon\varepsilon}^{*}-D_{\varphi\varphi}^{*}\right)-\sin^2\varphi D_{\varepsilon\varphi}^{*}\right)= \\
&-\frac{8\left[\pi^2+2\left(\pi^2-8\right)\varepsilon^2\right]}{\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}} \\
D_{YX}^{*} &= -S\left(\cos^2\varphi D_{\varepsilon\varphi}^{*}+\sin\varphi\cos\varphi\left(D_{\varepsilon\varepsilon}^{*}-D_{\varphi\varphi}^{*}\right)-\sin^2\varphi D_{\varphi\varepsilon}^{*}\right)= \\
&-\frac{8\left[\pi^2+2\left(\pi^2-8\right)\varepsilon^2\right]}{\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}} \\
D_{YY}^{*} &= -S\left(\cos^2\varphi D_{\varepsilon\varepsilon}^{*}-\sin\varphi\cos\varphi\left(D_{\varphi\varepsilon}^{*}+D_{\varepsilon\varphi}^{*}\right)+\sin^2\varphi D_{\varphi\varphi}^{*}\right)= \\
&\frac{2\pi\left[\pi^2+2\left(24-\pi^2\right)\varepsilon^2+\pi^2\varepsilon^4\right]}{\varepsilon\left(1-\varepsilon^2\right)^{\frac{1}{2}}\left[\pi^2+\left(16-\pi^2\right)\varepsilon^2\right]^{\frac{3}{2}}}
\end{aligned} \tag{25}$$

Further, the obtained results are in accordance with Ref. [5]. Therefore, the general transformation of the hydrodynamic journal bearing stiffness and dampness coefficients between polar and rectangular coordinates, as expressed by Eqs. (17) and (18), are verified.

4 Conclusions

The derived expression denotes the general transformation between polar and rectangular coordinates of journal bearing stiffness and dampness parameters. The validity of this study is confirmed with the use of short journal bearings. Further, this transformation is also suitable for long journal and finite journal bearings.

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