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Analysis of Outage Capacity of NOMA: SIC vs. JD

Shuang Chen, Kewu Peng, Huangpin Jin, and Jian Song*

Abstract: In fifth-generation wireless communication networks, Non-Orthogonal Multiple Access (NOMA) has attracted much attention in both academic and industrial fields because of its higher spectral efficiency in comparison with orthogonal multiple access. Recently, numerous uplink NOMA techniques have been proposed, some of which are based on Successive Interference Cancellation (SIC) and others on Joint Decoding (JD, or simultaneous decoding). In this study, we analyze the outage capacities of SIC and JD in the case of single-block transmission over a two-user Gaussian multiple-access channel with partial channel state information at transmitter from the perspective of information theory. Results of the analysis and numerals show that compared to SIC, JD can achieve a sum-rate gain of up to 10% or sum-power gain of 0.8 dB.

Key words: multiple access channel; successive interference cancellation; joint decoding; outage capacity

1 Introduction

In recent years, Non-Orthogonal Multiple Access (NOMA) has attracted much attention in both academic and industrial fields because of its superior capacity gain in comparison with orthogonal multiple access^[1-3]. In uplink NOMA systems, independent signals sent by multiple users are superimposed at the output of a Multiple-Access Channel (MAC), and all transmitted signals from these users are successively or simultaneously decoded at a receiver of the MAC. In the 1970s, Ahlswede^[4] determined the information-theoretical achievable Rate Region of MAC (MAC-RR), and in the past few decades, many coding and decoding techniques and schemes have

been proposed to approach the boundary of MAC-RR.

Successive Interference Cancellation (SIC) is the most well-known decoding technique. In SIC, a signal from a transmitter is decoded at a receiver first by treating the signals from others as noises. If the transmitting power and data rate for each user are carefully selected, all signals can be successfully decoded if this procedure is recursively performed. Theoretically, the use of SIC with time sharing provides the capability to approach the boundary of MAC-RR^[5]. Therefore, because of the capacity-approaching property of SIC, many SIC-based NOMA schemes have been recently proposed, e.g., successive interference cancellation amenable multiple access^[6]. However, as information is often transmitted block by block, time sharing cannot be adopted in a single-block transmission (also referred to as one-shot transmission).

In addition, recently, numerous Joint Decoding (JD, also referred to as simultaneous decoding) schemes have been proposed. For example, in a previous study^[7], an elementary signal estimator was introduced for achieving simple iterative multi-user detection without SIC, and in another study^[8], transmitted signals from multiple users were simultaneously decoded using sparse coding and an iterative multi-user message-passing algorithm.

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These capacity-approaching NOMA schemes have been mainly designed for use in scheduled uplink multiple-access systems, wherein the Channel State Information (CSI) is assumed to be perfectly known at the scheduler, and the scheduler can therefore select the optimal transmitting power and transmission rate to approach the boundary of MAC-RR. However, in the case of an imperfect Channel State Information at Transmitter (CSIT), which can be modeled as a block-fading channel^[9], MAC-RR obtained using a conventional Shannon (ergodic) capacity needs to be revised as an outage capacity, which is defined as the achievable rate region with a target outage probability^[10].

In this study, the outage capacities of SIC and JD decoders are analyzed and compared in the case of single-block transmission over a two-user Gaussian Multiple-Access Channel (GMAC) with imperfect CSIT. Results reveal that SIC is inferior to JD because of the probabilistic interference level in the case of imperfect CSIT. The remainder of this paper is organized as follows. In Section 2, the system model of a two-user GMAC with imperfect CSIT is reviewed, and in Section 3, the outage capacity of such a channel is discussed. In Section 4, the outage probabilities of SIC and JD are analyzed for a simple instance of a two-user GMAC via a numerical integral. Finally, Section 5 concludes the study.

2 Two-User GMAC

In this section, a typical communication scheme for a two-user GMAC is discussed in relation to analyzing and simulating a practical NOMA system with imperfect CSIT, wherein it is assumed that the communication system consists of two transmitters, e.g., terminal users and a receiver (e.g., a base station). The equivalent baseband channel model of a two-user GMAC is given by

$$Y_i = \sqrt{h_1} \cdot X_{1i} + \sqrt{h_2} \cdot X_{2i} + Z_i \quad (1)$$

where at time i , X_{1i} and X_{2i} are the transmitted symbols from two transmitters with an average power of P_1 and P_2 , respectively; $h_1, h_2 \in [0, +\infty)$ are the channel gains; Z_i is white Gaussian noise subject to a complex Gaussian distribution $CN(0, N_0)$; and Y denotes the received symbol superimposed with noise and signals from two users. In this paper, only perfect CSI at receiver is assumed.

Before transmission, the receiver selects objective

data rates according to the partial CSI, which is predicted using available information at the transmitter, and the data rate is broadcast to two transmitters. These two transmitters then send their own data to the receiver over the two-user GMAC. However, the CSI, h_1 and h_2 , in Eq. (1) is partial known due to the partial CSIT, and can thus be treated as random variables with a specific distributions. The probability density functions of h_1 and h_2 are written as $f_{h_1}(\cdot)$ and $f_{h_2}(\cdot)$.

3 Outage Capacity of Two-User GMAC

The outage capacity region of MAC, which is also referred as the outage rate region, can be divided into the common outage capacity and the individual outage capacity; this was previously defined^[10] as an achievable rate region with a target outage probability. The difference between the individual outage capacity and the common outage capacity is related to calculation of the outage probability. For example, in two-user multiple-access transmission, the common outage probability denotes the probability of success of both users. In other words, if the message of any user is lost then the transmission is treated as a transmission failure. Oppositely, the individual outage probability relates to the successful transmission probability of a specific user, no matter whether the transmission of the other user is a success or failure.

As the difference between individual outage capacity and common outage capacity is related only to the calculation of outage probability, in this section we only use the common outage capacity of MAC as an example.

3.1 Outage rate region

The common outage capacity of a MAC is defined as the achievable rate region with target outage probability,

$$C_{\text{MAC}}^{(p)} = \{(R_1, R_2) | \Pr((R_1, R_2) \in C_{\text{MAC}}) \geq 1 - p\} \quad (2)$$

where $\Pr(\cdot)$ denotes the probability function and p denotes the target outage probability. In this study, C_{MAC} denotes the capacity region of MAC with perfect CSIT and can be obtained by

$$C_{\text{MAC}} = \{(R_1, R_2) | R_1 \leq I(X_1; Y | X_2, H), \\ R_2 \leq I(X_2; Y | X_1, H), R_1 + R_2 \leq I(X_1, X_2; Y | H)\} = \\ \{(R_1, R_2) | R_1 \leq \log_2(1 + h_1 \cdot \frac{P_1}{N_0}),$$

$$R_2 \leq \log_2(1 + h_2 \cdot \frac{P_2}{N_0}),$$

$$R_1 + R_2 \leq \log_2 \left(1 + \frac{h_1 \cdot P_1 + h_2 \cdot P_2}{N_0} \right) \quad (3)$$

where $I(\cdot, \cdot | \cdot)$ denotes the conditional mutual information function; $H = (h_1, h_2)$ denotes the CSI; and R_k denotes the data rate of the k -th transmitter. As the CSI in Eq. (3), i.e., h_1 and h_2 , are random values, C_{MAC} are also random regions.

When JD is adopted at the receiver using the random coding technique, as previously determined in Ref. [4], the successful decoding condition is that of the transmission rate pair, $(R_1, R_2) \in C_{\text{MAC}}$, in the case of perfect CSIT. In other words, from an information-theoretic perspective, all rate pairs $(R_1, R_2) \in C_{\text{MAC}}$ can be approached via JD. Therefore, for all rate pairs $(R_1, R_2) \in C_{\text{MAC}}^{(p)}$ in the case of partial CSIT, the successful decoding probability is exactly the same as that in Eq. (2), and hence (R_1, R_2) can be achieved with the target outage probability, p .

However, in the case of an SIC decoder, the outage probability is increased due to the probabilistic interference level. For example, the receiver expects to firstly decode and cancel the signal from user 1, although it is possible that interference from user 2 could be worse due to an uncertain CSIT h_2 . In this case, the signal from user 1 cannot be decoded successfully, and therefore a decoding failure occurs.

To conduct analysis of the outage rate region when SIC is adopted, we use $C_{\text{SIC}(1,2)}$ to denote the achievable rate region, wherein the signal from user 1 is firstly decoded in the case of perfect CSIT. According to the information theory for MAC, $C_{\text{SIC}(1,2)}$ can be calculated as

$$\begin{aligned} C_{\text{SIC}(1,2)} &= \{(R_1, R_2) | R_1 \leq I(X_1; Y | H), \\ &R_2 \leq I(X_2; Y | X_1, H)\} = \{(R_1, R_2) | \\ &R_1 \leq \log_2 \left(1 + \frac{h_1 \cdot P_1}{h_2 \cdot P_2 + N_0} \right), \\ &R_2 \leq \log_2 \left(1 + \frac{h_2 \cdot P_2}{N_0} \right)\} \quad (4) \end{aligned}$$

Similarly, when the signal from user 2 is firstly decoded, the achievable rate region $C_{\text{SIC}(2,1)}$ can be calculated as

$$\begin{aligned} C_{\text{SIC}(2,1)} &= \{(R_1, R_2) | R_1 \leq \log_2 \left(1 + \frac{h_1 \cdot P_1}{N_0} \right), \\ &R_2 \leq \log_2 \left(1 + \frac{h_2 \cdot P_2}{h_1 \cdot P_1 + N_0} \right)\} \quad (5) \end{aligned}$$

As the decoder can select the decoding order dynamically, the successful decoding condition is that of $(R_1, R_2) \in C_{\text{SIC}} \triangleq C_{\text{SIC}(1,2)} \cup C_{\text{SIC}(2,1)}$, when the

receiver uses SIC to decode the signal. Therefore, the outage rate region of the SIC decoder, which denotes the achievable rate region with the target outage probability in relation to the SIC decoder, can be defined as

$$C_{\text{SIC}}^{(p)} = \{(R_1, R_2) | \Pr((R_1, R_2) \in C_{\text{SIC}}) \geq 1 - p\} \quad (6)$$

Because of the existence of $C_{\text{SIC}(1,2)} \subset C_{\text{MAC}}$ and $C_{\text{SIC}(2,1)} \subset C_{\text{MAC}}$, we are able to obtain $C_{\text{SIC}} \subset C_{\text{MAC}}$; this thus clarifies the existence of full inclusion of the outage rate region of the SIC decoder in the outage rate region of the JD decoder.

3.2 Outage power region

Although the outage rate region denotes the achievable rate region when transmission power is fixed, we also need to acknowledge the fixed objective transmission rate. When the objective transmission rate (R_1, R_2) is fixed, the outage power region is defined as the transmission power (P_1, P_2) required at the transmitter for the target outage probability,

$$\begin{aligned} Q_{\text{MAC}}^{(p)} &= \{(P_1, P_2) | \\ &\Pr((R_1, R_2) \in C_{\text{MAC}}(P_1, P_2)) \geq 1 - p\} \quad (7) \end{aligned}$$

where $C_{\text{MAC}}(P_1, P_2)$ denotes the capacity region of MAC and can be calculated by Eq. (3), wherein the transmission power of two users is represented by (P_1, P_2) .

Similar to our analysis of the outage rate region, we are able to obtain three conclusions, as follows. Firstly, for any transmission power $(P_1, P_2) \in Q_{\text{MAC}}^{(p)}$, the outage probability of a JD decoder is no more than p . Secondly, for an SIC decoder, the outage power region is defined as

$$\begin{aligned} Q_{\text{SIC}}^{(p)} &= \{(P_1, P_2) | \\ &\Pr((R_1, R_2) \in C_{\text{SIC}}(P_1, P_2)) \geq 1 - p\} \quad (8) \end{aligned}$$

where $C_{\text{SIC}}(P_1, P_2)$ denotes the capacity region of MAC with an SIC decoder, wherein the transmission power of the two users is (P_1, P_2) . Thirdly, the outage power region of the SIC decoder is fully included in that of the JD decoder.

4 Numeric Results

An accurate distribution of h_1 and h_2 was previously derived in Ref. [9] in the case of partial CSIT. However, for simplification without loss of generality, in this section we assume that $\sqrt{h_1}$ and $\sqrt{h_2}$ are subject to the Rician distribution, $\text{Rician}(s_1, \sigma)$ and $\text{Rician}(s_2, \sigma)$, where σ denotes the level of channel uncertainty, and s_1

and s_2 denote the partial CSIT. The Probability Density Function (PDF) of Rician(s, σ) can be written as

$$f_x(x) = \frac{x}{\sigma^2} \cdot \exp\left(\frac{-(x^2 + s^2)}{2\sigma^2}\right) \cdot I_0\left(\frac{x \cdot s}{\sigma^2}\right) \quad (9)$$

where $I_0(\cdot)$ denotes the modified Bessel function of the first kind with order zero.

To simulate a typical asymmetric two-user multiple system, as an example in this section we assume $s_1 = 1, s_2 = \sqrt{2}, \sigma = 0.1$, and thus the difference between the path losses of two users is 3 dB. In addition to the numeric results of common outage capacity, in this section the numeric results of individual outage capacity are also proposed as a supplement.

4.1 Outage rate region

The boundaries of the outage capacity region for the SIC and JD decoders are shown in the case of $P_1 = P_2 = N_0$ in Fig. 1a, and the maximum achievable sum rates with target outage probability, which restrict the throughput of a NOMA system, are calculated and plotted in Fig. 1b.

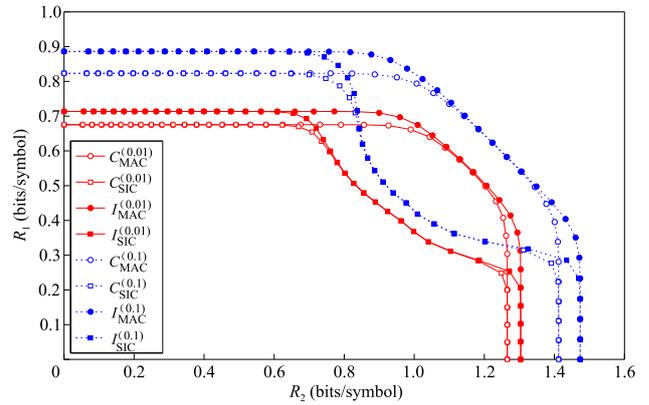
As shown in Fig. 1, not only the achievable rate region of SIC is different from that of JD in the case of imperfect CSIT, but the maximum achievable sum rate of SIC is also different from that of JD, and this occurs when either the common or individual outage is analyzed. Figure 1b plots in detail the differences between the maximum achievable sum rate of SIC and JD with target outage probability. As shown in the figure, if the common outage is analyzed, the achievable sum rate of SIC is 1.67 bits/symbol when the target outage probability is 10^{-1} , whereas that of JD is 1.84 bits/symbol. In addition, when individual outage is analyzed at a target outage probability of 10^{-1} , the achievable sum rate of SIC is 1.72 bits/symbol, while that of JD is 1.85 bits/symbol.

To conclude, SIC has a sum rate loss of up to 10% at a 10^{-1} target common outage probability when compared to JD, or a 6% sum-rate loss at a 10^{-1} target individual outage probability, and an even greater sum-rate loss when a lower target outage probability is used.

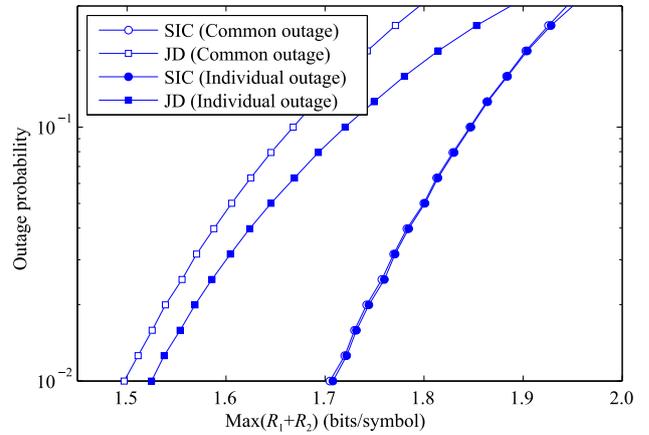
4.2 Outage power region

The boundaries of the outage power region of SIC and JD are shown in Fig. 2a for when the target rate pair $(R_1, R_2) = (1, 1)$, and the minimum achievable sum power with target outage probability is plotted in Fig. 2b.

Figure 2 therefore shows that in the case of an



(a) Boundaries of outage capacity region for SIC and JD decoders, where C denotes common outage capacity and I denotes individual outage capacity

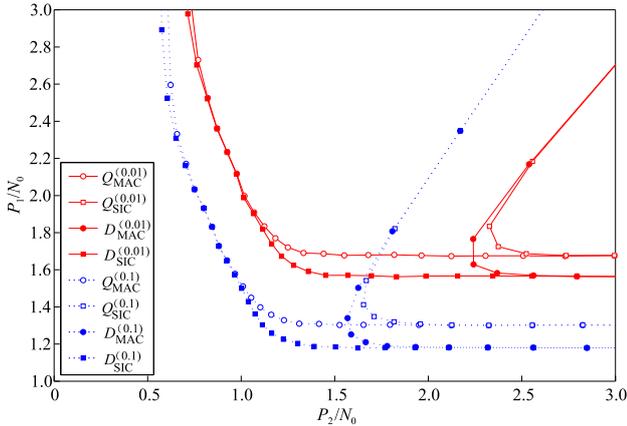


(b) Target outage probability with maximum achievable sum rates of SIC and JD

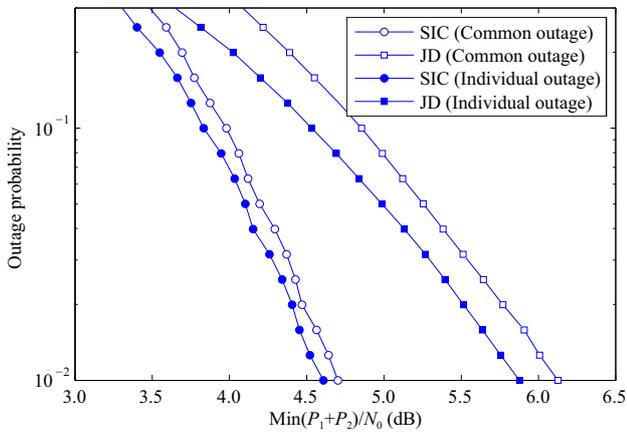
Fig. 1 Differences between outage capacity regions of JD and SIC.

imperfect CSIT, not only is the achievable power region of SIC much smaller than that of JD, but the minimum sum power required for target outage probability by SIC is also much larger than that of JD.

As shown in Fig. 2b, the minimum sum power required by the SIC decoder is $(P_1 + P_2)/N_0 = 4.8$ dB when the target outage probability is 10^{-1} in relation to common outage probability, or 4.5 dB for individual outage probability. For the JD decoder, the minimum sum power required is 4.0 dB in relation to common outage probability, or 3.8 dB in relation to individual outage probability. In conclusion, the results for the SIC scheme are approximately 0.8 dB inferior to that of the JD scheme, and there is a greater sum-power loss at a lower target outage probability. However, it should be noted that the decoding complexity of JD is much higher than that of SIC. For example, when QPSK constellation is adopted, the multi-user detection complexity of the n -user SIC decoder is about $O(4 \cdot n)$,



(a) Boundaries of outage power region of SIC and JD decoders, where Q denotes common outage power and D denotes individual outage power



(b) Target outage probability with minimum achievable sum power of SIC and JD

Fig. 2 Differences between outage power regions of JD and SIC.

while that of the n -user JD decoder is about $O(4^n)$. Hence, in a practical system with a huge number of users, it may not be practical for JD to be used by all users due to limited computing resources.

Therefore, as a trade-off between computing complexity and performance, a hybrid multi-user detection algorithm could be adopted. For example, multi-user detection is processed group by group, wherein users in the same group are decoded by the JD decoder, and the signal of the decoded group is then canceled from the received signal, by, for example, an SIC decoder.

5 Conclusion

In this paper, we analyze the outage capacities and outage power region of SIC and JD in the case of single-block transmission over a two-user GMAC with partial CSIT, from the perspective of information theory. To

present a typical case for such a channel, the outage rate regions and the outage power regions of SIC and JD decoders are shown and discussed. Furthermore, the maximum sum rates and minimum sum powers for target outage probability are also calculated and analyzed. Analysis and numeral results show that when transmission power is fixed, JD is able to achieve a maximum sum-rate gain of up to 10% at 10^{-1} target outage probability, compared to SIC. In addition, when the target transmission rate is fixed, the minimum sum transmission power required by SIC is about 0.8 dB inferior to that of JD for achieving the 10^{-1} target outage probability.

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